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MATHEMATICS

Polynomials

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Module 4

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Mathematics 9

Module 4

POLYNOMIALS

Alberta
EDUCATION

Mathematics 9
Student Module Booklet
Module 4
Polynomials
Learning Technologies Branch
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Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

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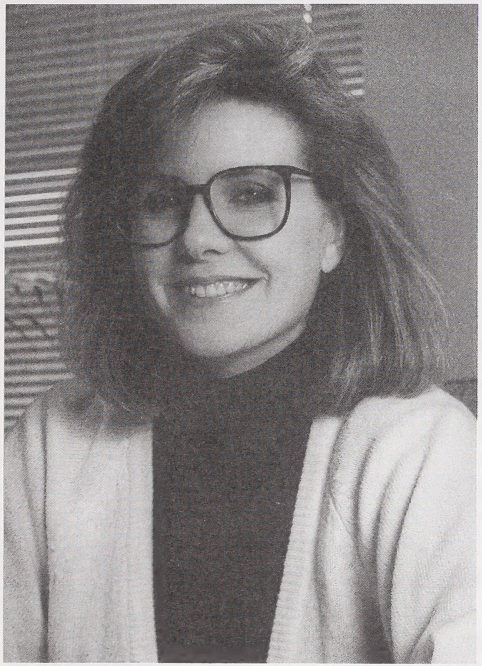
Welcome



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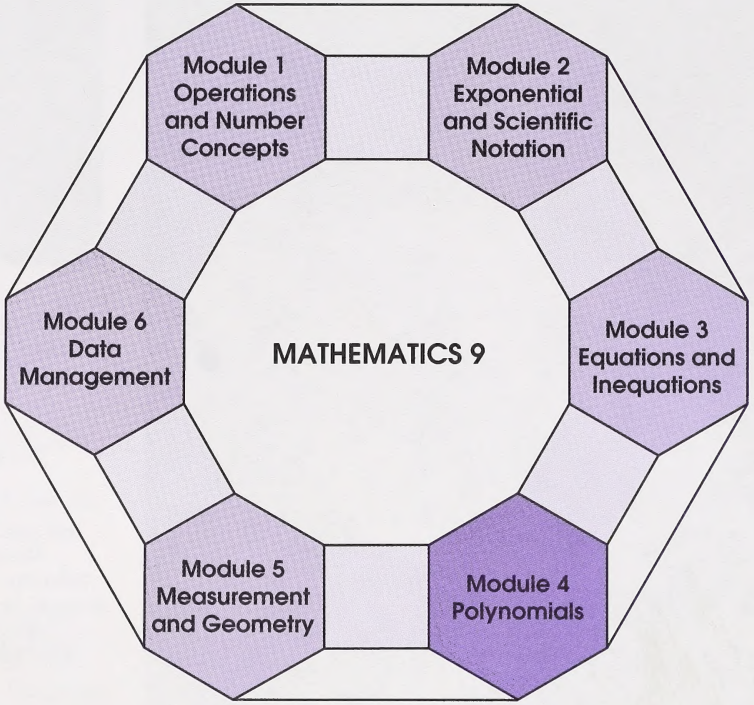
Welcome to Module 4. We hope you'll enjoy your study of Polynomials.

Mathematics 9 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

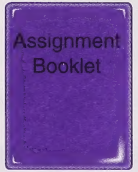
- Use computer software.

- Use a scientific calculator.

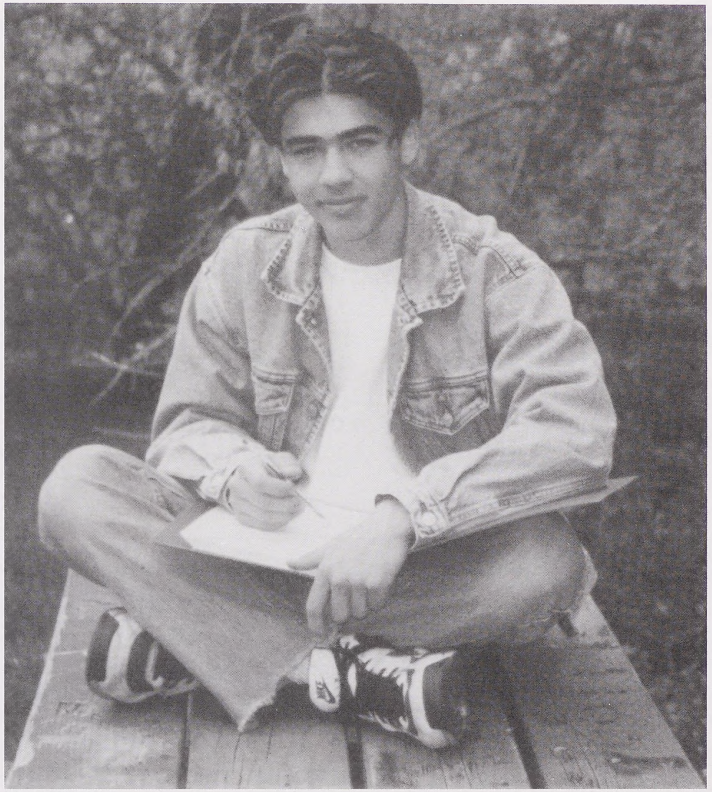
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

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Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

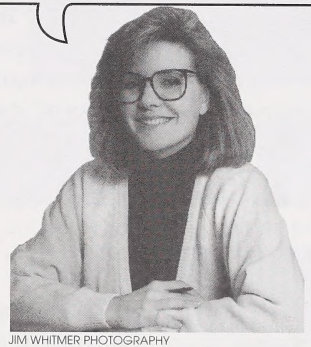
In Mathematics 9, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all students will be able to view the video programs and use a calculator, and that most students will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator or draw figures and graphs by hand.



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Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The icon is a cue that the problem will be related to the topic of the activity.

The icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in this module explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to this Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

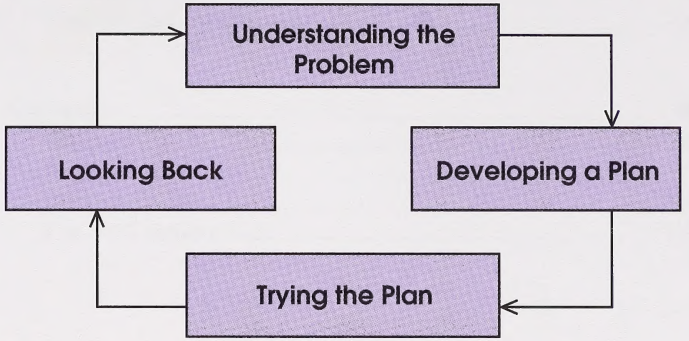
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

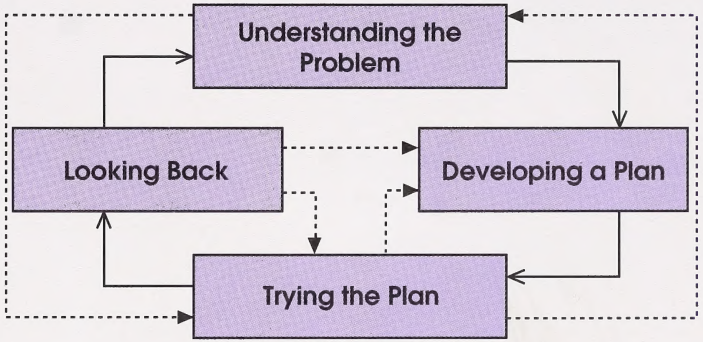
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



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Module Overview

Do you like to put jigsaw puzzles together? A large jigsaw puzzle contains many interconnecting pieces and can be difficult to put together. But, if you pay attention to the various patterns and colours of the picture, you can fit the pieces together.

Similarly, the study of algebra is composed of many interconnecting parts. You have completed some of the pieces of the algebra puzzle in previous mathematics courses and in Module 3.

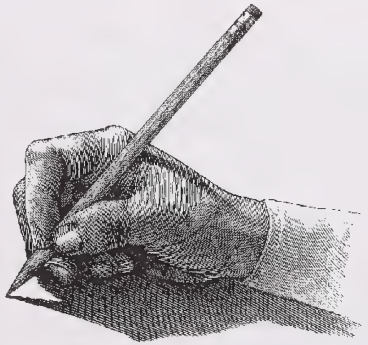
In Module 4 you will study more about algebra as you examine polynomials. You will discover what polynomials are, their functions, and how to work with them. You will model polynomials using algebra tiles; add, subtract, multiply, and divide various types of polynomials; and use a number of ways to factor and simplify polynomials (similar to what you did with whole numbers in Module 1).

Evaluation

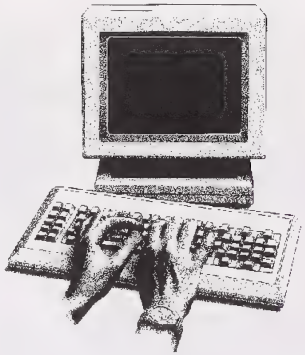
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete two section assignments and a final module assignment. The mark distribution is as follows:

Section 1 Assignment	**45 marks**
Section 2 Assignment	**20 marks**
Final Module Assignment	**35 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Operations with Polynomials

A good house can only be built on a properly constructed foundation. If the foundation is not laid properly, cracks will soon appear in the walls, doors and windows will not fit properly, and numerous other faults will surface.

Similarly, you cannot build a good knowledge of algebra, particularly polynomials, without a good grasp of the basics. You must lay a solid foundation of basic ideas concerning polynomials before you can carry on with operations involving polynomials.

The foundation for your work with polynomials will be done in the first activity of this section. For the remainder of this section, you will build a framework by adding and subtracting polynomials and becoming familiar with ways of multiplying various types of polynomials. Dividing polynomials by monomials will be examined to complete the framework. Becoming confident with the computation of polynomials through modelling will provide a sturdy framework for further explorations with polynomials in Section 2.

Activity 1: Interpreting Polynomials

Before getting your driver's licence in Alberta, you must know what various road signs mean. Similarly, before proceeding in your work with polynomials, you should become familiar with their various parts, their meanings, and how to evaluate them—building that firm foundation in algebra.

In this activity you will identify the various terms involved with polynomials. You will also model polynomials using algebra tiles, and you will classify and evaluate polynomials.

In Module 3, you examined some of the vocabulary of algebra. You will now expand that base as you study some new terminology.

The following is an example of a **polynomial**.

$$4n^2 + mn + 8$$

A polynomial is made up of several parts. Each of the groups of numbers and/or letters separated by the addition signs is called a **term**. For example, the terms in the preceding example of a polynomial are $4n^2$, mn, and 8.

Each term is made up of a number factor, called the **numerical coefficient**, and a variable factor (or factors), called the **literal coefficient**. A term that consists of a number by itself is called a **constant**.

In the first term $(4n^2)$,

- 4 is the numerical coefficient.
- n^2 is the literal coefficient.
- n is the variable.

In the second term (mn),

- 1 is the numerical coefficient.
- mn is the literal coefficient.
- m and n are the variables.

In the third term (8),

- 8 is a constant.

Terms with identical literal coefficients are called **like terms** (recall from Module 3). Both the variables and exponents must be the same in like terms. Examples of like terms are $3a$ and $2a$, n^2 and $-4n^2$, and a^2b and $-3a^2b$.

1. Identify the numerical coefficients, literal coefficients, and variables for each of the following terms.

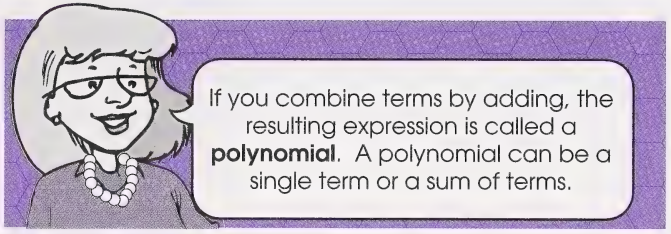
a. $7ab$ **b.** $-8y^3$ **c.** $4xyz$

d. $-6z$ **e.** $2x^2y^3$ **f.** $\frac{ab}{2}$

2. Group the like terms in the following list.

- m^2n
- $2m^2n$
- $-5a$
- $2mn$
- -9
- $4a$
- $-3a^2b$
- 6
- $-2ab$
- $\frac{1}{2}m^2n$
- $-5ab$
- $-12a^2b$
- $-8m^2n$
- mn
- $16a$

Check your answers by turning to the Appendix.

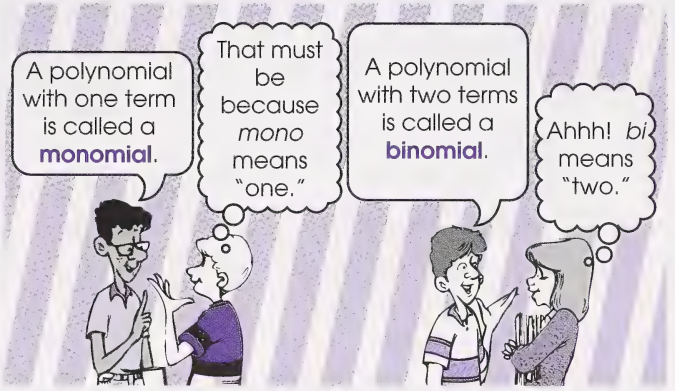


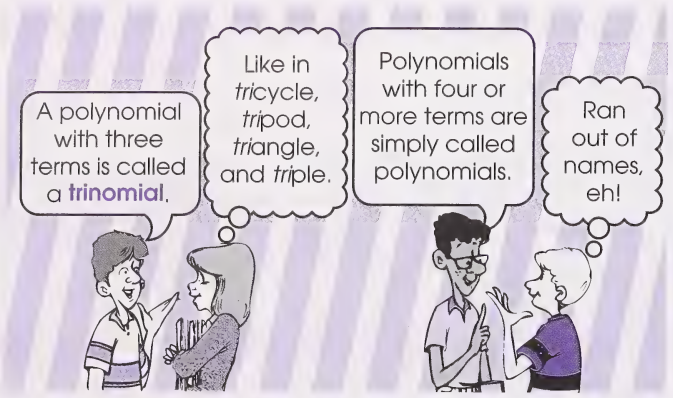
Note: In a polynomial, the exponents of the literal coefficients must be whole numbers, and the literal coefficients must be in the numerator.

Some examples include the following:

- $4b$
- $4a-2b$
- $1.4a^2-2$
- $6ab$
- $\frac{2}{5}a-\frac{1}{4}$
- $7a^2+8a-6$
- $7a^3+4a^2+3a-2$
- ab^2
- $a^2b+ab+a-b+7$

There are special names for polynomials that have one, two, or three terms.





Example 1

Which of the following are monomials? Which are binomials? Which are trinomials?

- $2a+3b+4c+5d$
- $7m^2-6m+1$
- $4c+2d-e$
- $6a^{-2}+\frac{4}{a}-2$
- $4ab+c$
- $x+y+z$
- $4+3a$
- $8m^2-6$
- $3abc$
- $-4y$
- 14

Solution

The monomials are $3abc$, $-4y$, and 14.

The binomials are $4ab+c$, $4+3a$, and $8m^2-6$.

The trinomials are $7m^2-6m+1$, $4c+2d-e$, and $x+y+z$.

3. Why are two of the expressions in Example 3 not included in the solution?

4. Indicate whether each of these expressions is a monomial, binomial, or trinomial.

a. -7 **b.** $4xyz$

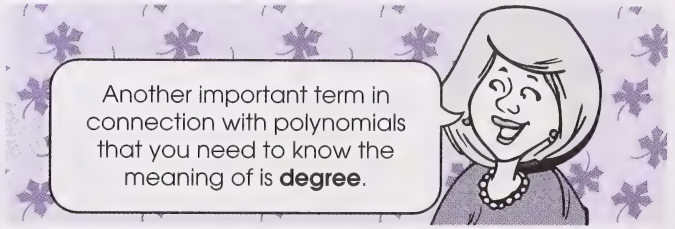
c. $g+h$ **d.** $4.5a-b+c$

e. $x^2+x^2y+x^2y^2$ **f.** $24z-13y+3x$

g. $8-2a$ **h.** $1.3ab^2c^3d^4$

i. $\frac{a}{2}+\frac{b}{3}+\frac{c}{4}$

Check your answers by turning to the Appendix.



The **degree of a monomial** is the sum of the exponents of all of the variables in the term. The **degree of a polynomial** is the degree of the term that has the largest sum of exponents.

Example 2

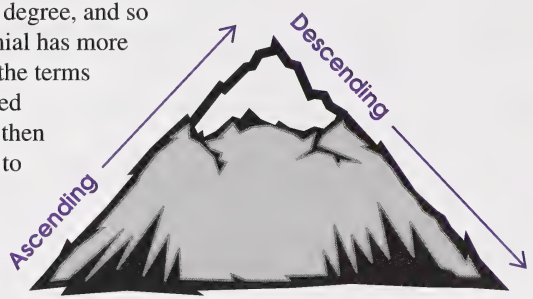
What is the degree of each of these expressions?

- $8n^3$
- $4a^2b^3c^4$
- $7a^2+7b^3$
- x^4+7x^3-8
- $2x^2y^4+3x^2y^3-6xy^5$
- $3m^3n^2-8mn^6+7m^3n^3$
- 9

Solution

Expression	Degree
$8n^3$	3
$4a^2b^3c^4$	9 (2+3+4)
$7a^2+7b^3$	3
x^4+7x^3-8	4
$2x^2y^4+3x^2y^3-6xy^5$	6 (2+4 or 1+5)
$3m^3n^2-8mn^6+7m^3n^3$	7 (1+6)
9	0

The terms of a polynomial are usually written so that the degree of the terms are in **descending order**. (Sometimes they are written in **ascending order**.) If the polynomial has only one variable, the term with the highest degree would be written first, followed by the term with the second highest degree, and so on. If the polynomial has more than one variable, the terms will first be arranged alphabetically and then in order according to the first variable (then the second or third as necessary).



Example 3

Arrange the following polynomials in descending order.

- $3+5b$
- $8+3c+5c^3-2c^2$
- $4ab-2a^2b+3ab^2$
- $6x^2y-8x^2y^3+2x^2y^4-x^2y^2$

Solution

$5b+3$

The term with the variable comes first because the constant has degree zero.

$5c^3 - 2c^2 + 3c + 8$
There is only one variable, so the terms are written in order from highest degree to lowest.

$-2a^2b + 3ab^2 + 4ab$
The terms are arranged according to the variable a (highest to lowest degree) first; then when the a has the same exponent, the terms are arranged according to the variable b (highest to lowest degree).

$2x^2y^4 - 8x^2y^3 - x^2y^2 + 6x^2y$
All the terms have the same exponent for the variable x; so, they are arranged according to exponents for the variable y.

5. State the degree for each term.

a. $6a^3$ **b.** $-7ab^2$ **c.** abc
d. -12 **e.** $8a^2b^3c$ **f.** $1.4xy^2z$

6. State the degree for each polynomial.

a. $7a + 2b^2$
b. $x^2 + x - 1$
c. $-2m^3n + 4m^2n - 2m$
d. $7y + 2$
e. $4x^2y + 6x^2y^3 - 14xy + 2x^3y^2$
f. $6 - 2n^3 + 4n + 5n^2$

7. Write each of the polynomials in descending order.

a. $7a + 8 + 3a^2$
b. $2b^3 - 4b + 6 + 8b^2$
c. $16 - 2c$
d. $4m^3n^2 - 2mn + 6m^2n - 4$
e. $9 - 2x^3y^4 + 3x^3y^2 - x^2y^2 + 3xy$
f. $4xyz + x^3y^2z - 7x^2y^2z^2 + 2x^4y^2z$

Check your answers by turning to the Appendix.

Classifying

Polynomials are traditionally classified according to either degree, number of terms, or number of variables.

Example 1

How would you classify the polynomial $4x^2 + 2x - 6$?

Solution

$4x^2 + 2x - 6$ is a second-degree trinomial in one variable.

Example 2

Classify the polynomial $5x^3y^2 - 6xy^2$.

Solution

$5x^3y^2 - 6xy^2$ is a binomial of degree 5 in two variables.

8. Copy and complete the following chart in your notebook. Then classify each polynomial according to the criteria in each heading.

Polynomial	Number of Terms (name)	Degree	Number of Variables
$-2+6a+8a^2$			
$-3b+7b^3$			
16			
$2d$			
$7y-4xy+2x^2y^3$			
$6mn+10m^2n^4$			
$-1+y-2y^2+3y^3+4y^4$			

9. What do you notice about every polynomial in question 8?

10. Write a polynomial that has the described characteristics.

a. a monomial of degree 4 in two variables

b. a second-degree trinomial in one variable

c. a first-degree binomial in one variable

d. a polynomial of degree 5 in one variable

e. a trinomial of degree 6 in two variables

Check your answers by turning to the Appendix.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

11. One day, Mr. Wilkinson had a craving for chocolate, so he bought a box of chocolates on the way home from work. Before he knew it, he had eaten one-sixth of them. He was feeling a bit guilty when he got home, so he offered the rest to his wife, Irene. She thanked him and, while watching television that evening, ate one-fifth of what was left. Their oldest son, Gary, saw the open box when he got home and ate one-quarter of what was left. Janine, the Wilkinsons' daughter, nibbled away at one-third of those left when she got up the next day. Her younger brother, Anthony, caught her licking her fingers afterwards and promptly ate one-half of the remaining chocolates. By the time Mrs. Wilkinson got back to the box of chocolates, there were only four left. How many chocolates were originally in the box?

Solve this problem in two different ways.

Check your answer by turning to the Appendix.

Modelling Polynomials

Models are used to show off new clothing designs to prospective buyers. They help to give the buyers a better picture of what the clothing looks like on a person.

Models can also be very useful in helping you understand algebra. The work that you will do is similar to what you did in Module 3 (involving cylinders and counters), except now you will use algebra tiles.

You will start your study of modelling of polynomials by watching a video that introduces the concept of algebra tiles. You will need the algebra tiles from the back of the Appendix to answer the questions on the video. You may wish to glue the tiles onto heavier paper (such as construction paper, boxboard, or cardboard). You do not need to do the parts involving the binomial grid.

Watch the video entitled *Joe Sceptic*; then do the video assignment. This video is available from the Learning Resources Distributing Centre.

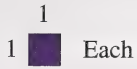
Check your answers to the video assignment by viewing the video.

12. What is the relationship between the length of an x-tile and the length of a side of an x^2-tile.

13. What is the relationship between the length of a side of a 1-tile and the width of an x-tile?

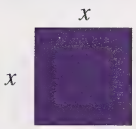
Check your answers by turning to the Appendix.

The following are the different types of tiles that you saw in the video.



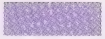
Each tile like this represents 1.

Each tile like this represents x (or any other variable).



Each tile like this represents x^2.

Each tile like this represents -1.

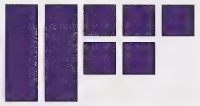


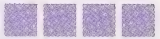
Each tile like this represents $-x$.

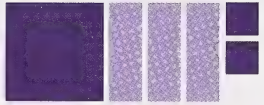
Each tile like this represents $-x^2$.

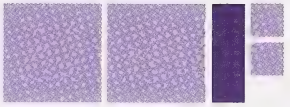
Use the preceding information to help answer questions 14 and 15.

14. Write the expression represented by each of these groups of tiles. **Note:** You can use any variable to write the expression.

a. 

b. 

c. 

d. 

e.

15. Use algebra tiles to model each of the following expressions.

a. $2a^2 - 3a + 1$ **b.** $b^2 + b - 6$

c. $-3c^2 + 4c + 2$ **d.** $d^2 - 2d + 5$

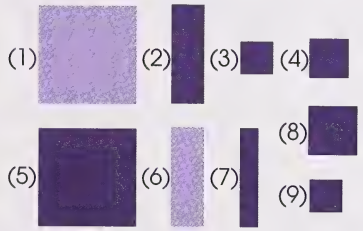
e. $e^2 + 3e - 4$

Check your answers by turning to the Appendix.

Like Terms

You discovered in Module 3 that like terms are important in algebra. When modelling polynomials using algebra tiles, like terms can be represented by congruent shapes (shapes that are the same size and shape).

16. Use the shapes given to answer the questions that follow.



a. Which of the shapes shown are congruent? Explain.

b. Does the colour affect whether or not the shapes are congruent?

Check your answers by turning to the Appendix.

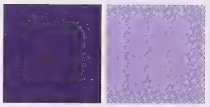
Each of the congruent shapes are **like** shapes and represent like terms.

Representing Zero

You used the concept of zero when solving equations in Module 3. The concept of zero is important in operations with polynomials as well. You can represent or model zero by using the appropriate tiles.

17. What does each group represent?

a.



b.

c.

d.

18. What is each of the following equal to?

a. $4x^2$ and $-4x^2$
b. $7x$ and $-7x$
c. -1 and 1

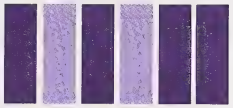
Check your answers by turning to the Appendix.

Zero can be represented by using the same number of positive and negative (or opposite) like tiles. You can refer to this as the **zero principle**.

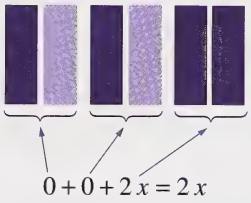
The following examples show various combinations of tiles including some zero groups.

Example 1

What do these tiles represent?

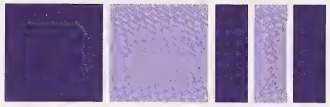


Solution

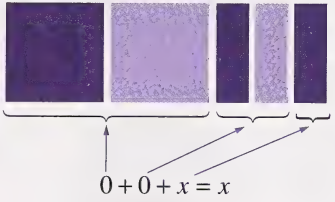


Example 2

What do these tiles represent?

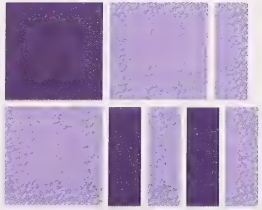


Solution

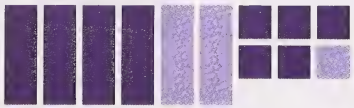


19. What does each group of tiles represent?

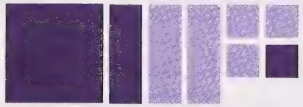
a.



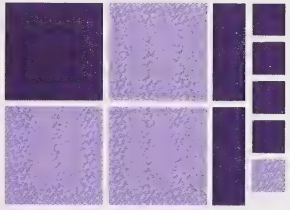
b.



c.



d.

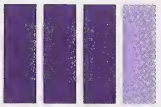


20. Use algebra tiles to represent zero in three different ways.

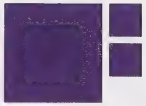
21. What tiles can you add to each group to make the entire group zero?

a.

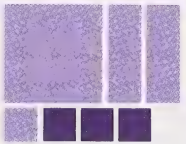
b.



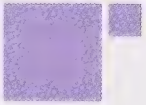
c.



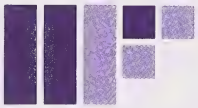
d.



e.



f.



22. Use algebra tiles to represent each expression; then simplify by collecting like shapes and eliminating the ones that are equivalent to zero.

a. $4a^2 + 2a + 3 - 2a^2 - 3a + 1$

b. $3b^2 + 5 + b^2 + 3b - 7$

c. $2c^2 - 4c + 1 - 5c^2 + c - 4$

d. $d^2 - d + 7 + 2d^2 + 3d - 2$

23. When collecting the like shapes in question 22, what do you do to simplify?

24. Simplify each of the following **without** using algebra tiles.

a. $7m+2m-3m$

b. $8n^2-2n^2+3n^2$

c. $8y+7+4y-5$

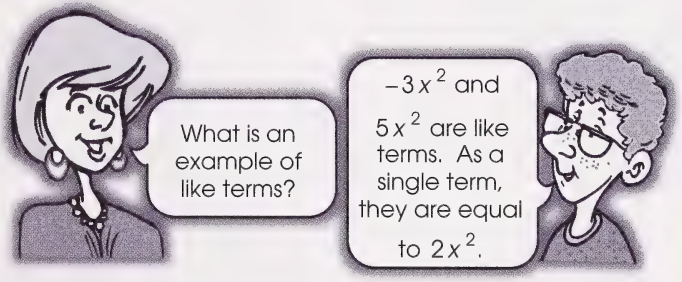
d. $3x^2+2x+4x^2-5x+1$

e. $b^2-2b+4-3b^2+3b-1$

25. Explain how you can simplify expressions containing like terms without using algebra tiles.

Check your answers by turning to the Appendix.

Like terms are combined or made into a single term by adding their numerical coefficients.



Example 4

Simplify the expression $7y^2+8y-4+2y^2-6y+3$ by collecting like terms.

Solution

$$7y^2+8y-4+2y^2-6y+3=7y^2+2y^2+8y-6y-4+3$$
$$=9y^2+2y-1$$

$7+2$ $\quad$ $8-6$ $\quad$ $-4+3$

26. Simplify each expression by combining like terms.

Note: m means $1m$ and $-m$ means $-1m$.

a. $2m+m-m-5$

b. $3y^2+4y-2y^2+6y-1$

c. $x^2-y^2+4x+3y-8+x^2-y^2+2x+3$

d. $4a^2b+3a^2+7-2b^2+a^2b+4b^2-3$

e. $t+t+8-t^2+4t-6+3t^2-2$

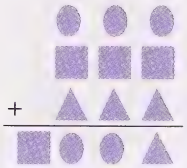
27. Find the puzzle "How Does a Hawaiian Baritone Laugh?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

Check your answers by turning to the Appendix.

Now Try This

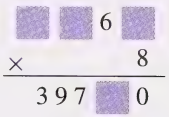
Use one of the problem-solving strategies to solve the following problem.

28. a. Each shape in the sum given represents a certain digit. Determine the missing digits that will make the sum true.



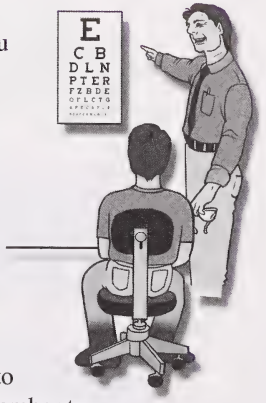
b. Determine the missing digits that will make the product true.

Note: Each ■ may represent a different digit.



Check your answers by turning to the Appendix.

Evaluating Polynomials



Optometrists can evaluate the condition of your eyes and determine whether or not you need glasses. They are able to do this by comparing your vision to a norm through a series of tests.

Polynomials can be evaluated if you know what numbers the variables are to be replaced by. The procedure is the same as the one you studied in Module 3 for evaluating expressions. Substitute the indicated number into the polynomial in place of the variable; then use a calculator to carry out the indicated computations. Remember to follow the rules for order of operations.

Example 1

Evaluate the trinomial $8y^2 - 2y + 5$ if $y = 2$.

Solution

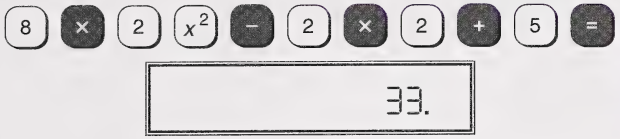
Write the polynomial.

$$8y^2 - 2y + 5$$

Substitute 2 for y.

$$8(2)^2 - 2(2) + 5$$

Do the computation using a calculator.



The value of $8y^2 - 2y + 5$, when $y = 2$, is 33.

Example 2

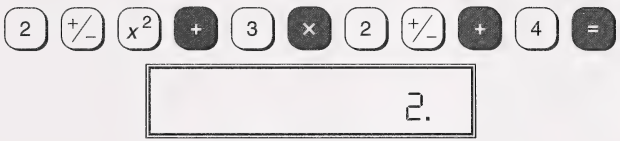
Evaluate the polynomial $5n^2 - 2n + 4n - 3 + 7 - 4n^2 + n$ if $n = -2$.

Solution

First, simplify the polynomial by collecting like terms.

$$\begin{aligned} & 5n^2 - 2n + 4n - 3 + 7 - 4n^2 + n \\ &= 5n^2 - 4n^2 - 2n + 4n + n - 3 + 7 \\ &= n^2 + 3n + 4 \end{aligned}$$

Now, substitute -2 for n into the simplified polynomial.



The value of $5n^2 - 2n + 4n - 3 + 7 - 4n^2 + n$, when $n = -2$, is 2.

Use a calculator to answer questions 29 and 30.

29. Evaluate the following polynomials if $m = 3$ and $n = -2$.

a. $2m^2 - 3m + 4$ **b.** $2mn + 4n + 8$

c. $4 - 2mn - 5n^2$ **d.** $m^2n - 2mn^2 + 3mn - 4$

30. Evaluate the trinomial $3x^2 - 2x + 5$ using the given replacement for x.

a. $x = -2$ **b.** $x = 3$

31. Simplify each expression; then evaluate if $a = 1$, $b = 2$, and $c = -3$. **Note:** It may be easier to do this using paper and pencil rather than using a calculator.

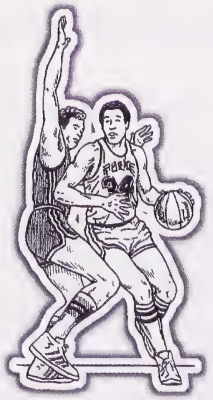
a. $4a + 7 - a$

b. $2a^2 - 3b^2 + 2a - 3b^2$

c. $8a^2b + 2ab - a^2b + ab + 6$

d. $a^3 + b^2 + c + 4a^3 - 3b^2 - 5c + 8$

32. The Flying Eagles scored $4a+35b+17c$ points in their basketball game against the Leaping Lions, who scored $6a+40b+10c$ points. In basketball, you score three-point field goals (a), two-point field goals (b), and one-point free throws (c). Which team won this game and what was the final score?



33. The Pacific Coast Hockey League instituted a new system for obtaining points based on the following results:

- 3 points for a win (w)
- 2 points for an overtime tie (t)
- 1 point for an overtime loss (ℓ)

a. Write an expression to indicate the total points a team obtains for the season.

b. Find the season total points for each of the following teams from the league.

Team	w	t	ℓ	Total Points
Victoria	40	3	2	
Spokane	42	2	3	
San Diego	27	4	5	
Portland	47	4	3	
Seattle	16	3	5	

c. Which team finished in first place?

d. If they play a 72-game schedule, how many games did Portland lose in regulation time?

34. Fill in the shapes (●, ■, ▲) in each polynomial with the correct numerical coefficient so that each polynomial equals $4x-3y+2z$ when written in simplest form.

a. $7x+\blacksquare y-\blacktriangle z-5y+4z-\bullet x$

b. $\bullet x+2y-3z-\blacksquare y+\blacktriangle z-5x$

c. $x+y+z+\bullet x-\blacksquare y+\blacktriangle z$

d. $-3x+\blacksquare y-\blacktriangle z+\bullet x-7y+8z$

35. You are asked to simplify the polynomial $4d+8-2d-1+6d-4$. How could you use the value of d (for example, $d=1$) to check your simplification of polynomial? Check it out.

36. Find the puzzle "What Do You Call Two Railroad Trains After a Head-on Collision?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

Check your answers by turning to the Appendix.

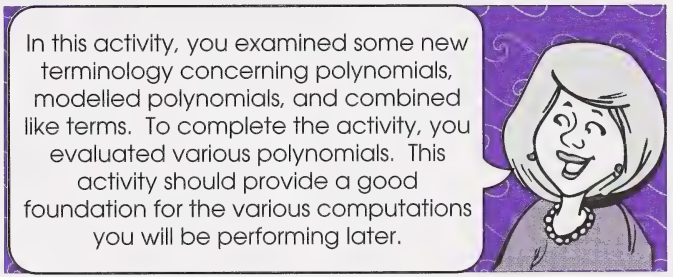
Now Try This

Use one of the problem-solving strategies to solve each of the following problems.

37. Barbara bought balloons for the school dance. She spent \$6.15 for 23 balloons. If black balloons cost \$0.65 each, red balloons cost \$0.25 each, and white balloons cost \$0.15 each, then how many of each kind did she buy?

38. Beginning at the P and ending at the S, how many paths can you find that spell the word POLYNOMIALS?

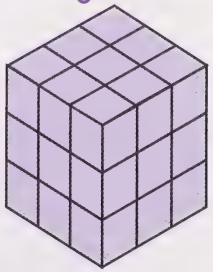
Check your answers by turning to the Appendix.



Activity 2: Adding and Subtracting Polynomials

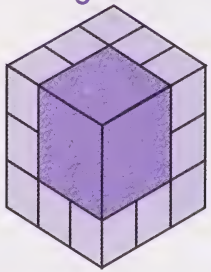
How many cubes are there in each of the following figures?

Figure A



(27)

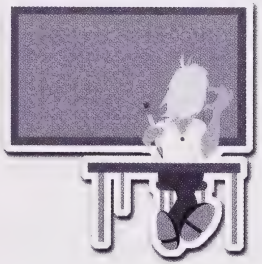
Figure B



(19 small ones + 1 large one)

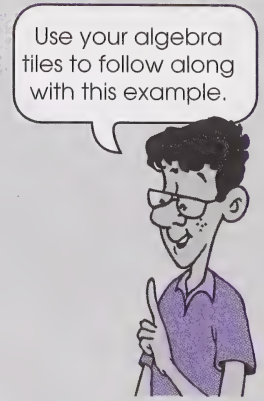
You can really only add those cubes that are identical. The same concept applies to addition (and subtraction) of polynomials.

In this activity you will add and subtract polynomials with the aid of the algebra tiles used to model polynomials in Activity 1.



Adding Polynomials

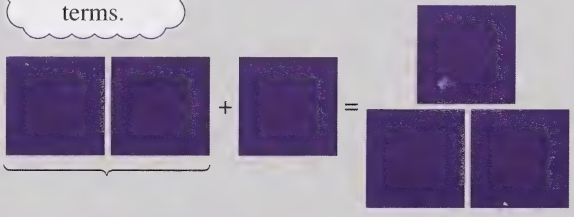
A rectangle has an area of $2x^2$ square units, and a square has an area of x^2 square units. To find the combined area, simply add the two areas.



$A = 2x^2$ $A = x^2$

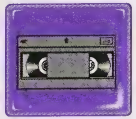
Use the algebra tiles to model this addition. To add the two areas, combine the tiles that are the same size and shape.

Simplify by combining like terms.



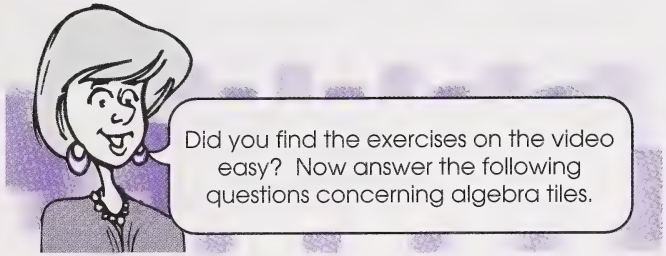
$2x^2$ x^2 $3x^2$

Therefore, $2x^2 + x^2 = 3x^2$.



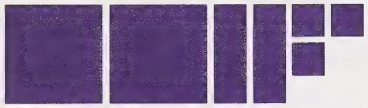
Watch the video entitled *Tilly Tile*. Stop at the end of the addition of polynomials (first ten minutes). Use the algebra tiles to do the video assignment. This video is available from the Learning Resources Distributing Centre.

Check your answers to the video assignment by viewing the video.



Use the following tile diagrams to answer questions 1 to 3.

Tile Diagram A



Tile Diagram B

1. **a.** How many tiles of each kind (x^2-tiles, x-tiles, and 1-tiles) are in Tile Diagram A?

b. How many tiles of each kind are in Tile Diagram B?

2. How many tiles of each kind are in the two tile diagrams combined?

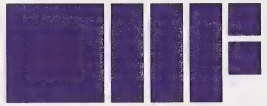
3. Write a polynomial that represents the following:

a. Tile Diagram A
b. Tile Diagram B
c. Tile Diagrams A and B combined

4. Find the sums of the algebra tiles in each pair of diagrams. Write the equivalent addition algebraically below each set of diagrams.

a.

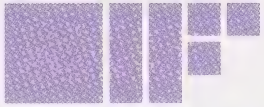
A



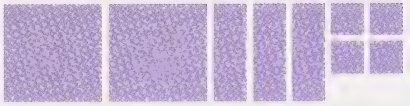
B

b.

A

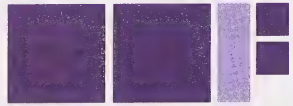


B

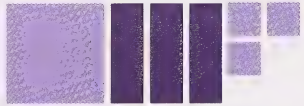


c.

A

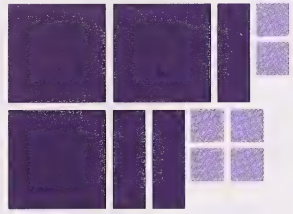


B

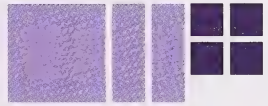


d.

A



B



5. Refer to the algebraic additions in question 4, and state how you can add each pair of polynomials without using algebra tiles.

Check your answers by turning to the Appendix.

To add two polynomials without using algebra tiles, combine the like terms and add their coefficients.

The following example illustrates addition of polynomials both using the algebra tiles and algebraically.

Example 1

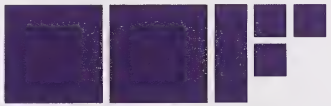
Find the sum of $\left(2x^2+x+3\right)$ and $\left(-x^2+3x-5\right)$.

Solution

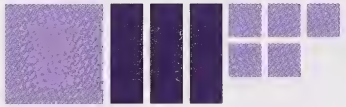
Method 1

Step 1: Model each polynomial.

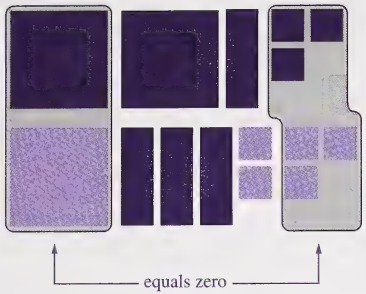
$2x^2+x+3$



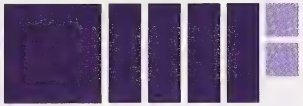
$-x^2+3x-5$



Step 2: Use the zero principle to simplify.

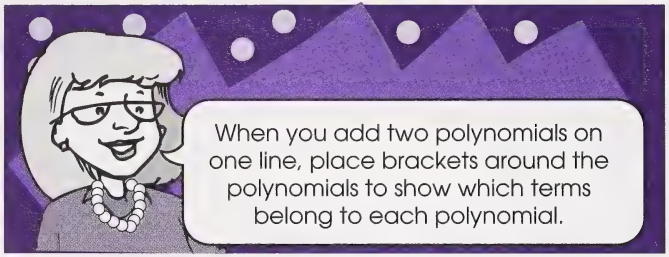


Step 3: Combine like terms.



$x^2 + 4x - 2$

Therefore, $\left(2x^2 + x + 3\right) + \left(-x^2 + 3x - 5\right) = x^2 + 4x - 2$.



Method 2

Write the polynomials on one line.

$$\begin{aligned}\left(2x^2 + x + 3\right) + \left(-x^2 + 3x - 5\right) &= 2x^2 + x + 3 - x^2 + 3x - 5 \\ &= 2x^2 - x^2 + x + 3x + 3 - 5 \\ &= x^2 + 4x - 2\end{aligned}$$

Add the like terms by adding their coefficients.

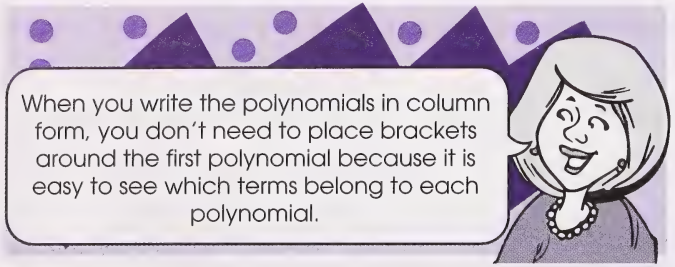
Method 3

Write out the polynomials one below the other.

$$\begin{array}{r} 2x^2 + x + 3 \\ +\left(-x^2 + 3x - 5\right) \\ \hline x^2 + 4x - 2 \end{array}$$

$2 + (-1) \qquad 1 + 3 \qquad 3 + (-5)$

Therefore, $\left(2x^2 + x + 3\right) + \left(-x^2 + 3x - 5\right) = x^2 + 4x - 2$.



Example 2 illustrates the addition of two polynomials using algebra only.

Example 2

Add $\left(-2x^2+3x-5\right)$ and $\left(4x^2+8\right)$.

Solution

Method 1

Collect like terms and simplify.

$$\begin{aligned}\left(-2x^2+3x-5\right)+\left(4x^2+8\right) &= -2x^2+3x-5+4x^2+8\\ &= -2x^2+4x^2+3x-5+8\\ &= 2x^2+3x+3\end{aligned}$$

Method 2

Use column form.

$$\begin{array}{r} -2x^2+3x-5 \\ +\left(4x^2\qquad +8\right) \\ \hline 2x^2+3x+3 \end{array}$$

Verification

You can verify your solution by substituting any value for the variable in both the original polynomial and the new polynomial. This is an application of evaluating polynomials which you performed in Activity 1 of this section.

To verify the addition of the polynomials in this example, pick a number and substitute it for x. Try $x=2$.

LS	RS
$\left(-2x^2+3x-5\right)+\left(4x^2+8\right)$	$2x^2+3x+3$
$=\left[-2(2)^2+3(2)-5\right]+\left[4(2)^2+8\right]$	$=2(2)^2+3(2)+3$
$=\left[-2(4)+6-5\right]+\left[4(4)+8\right]$	$=2(4)+6+3$
$=(-8+6-5)+(16+8)$	$=8+6+3$
$=-7+24$	$=17$
$=17$	

LS = RS

Therefore, $\left(-2x^2+3x-5\right)+\left(4x^2+8\right)=2x^2+3x+3$.

6. Use algebra tiles to add the following polynomials. Show the equivalent algebraic solution for each.

a. $(2x^2+3x-5)+(3x^2-x+2)$

b. $(3y^2-4y+1)+(-2y^2+3y+4)$

c. $(4a^2-5a)+(2a^2+3a-2)$

d. $(y^2+6y+5)+(3y^2-8)$

e. $(-3b^2+5b+1)+(b^2-3b+4)$

7. Determine the sum of each pair of polynomials. Remove the brackets, collect the like terms, and simplify.

a. $(a^2+2a+7)+(4a^2-5a+3)$

b. $(3b^2-9b+1)+(-5b^2+2b+3)$

c. $(4c^4+5c^2+7c+1)+(3c^4+2c^2-6c-4)$

d. $(d^3+d^2-4+3d)+(2d^3-5d+6d^2)$

e. $(m^2+7m-3)+(2m^2-5)+(-5m^2+m+1)$

8. Add these polynomials in column form.

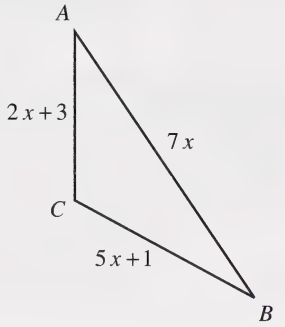
a.
$$\begin{array}{r} 2x^2+7 \\ +\ (3x^2-4) \\ \hline \end{array}$$

b.
$$\begin{array}{r} 3a^2+5a+6 \\ +\ (-4a^2+2a-3) \\ \hline \end{array}$$

c.
$$\begin{array}{r} 8n^2-5n+7 \\ +\ (-3n^2\quad -4) \\ \hline \end{array}$$

d.
$$\begin{array}{r} 3m^3+2m^2-4m+8 \\ +\ (\ m^3\quad +7m-6) \\ \hline \end{array}$$

9. a. Write an expression for the perimeter of ΔABC.



b. Write the expression in simplest form.

c. If $x = 3\,\text{cm}$, what is the perimeter?

10. The perimeter of a plot of land is given by the expression $(2x+3)+(3x-2)+(x+8)$.

a. Write the expression in simplest form.

b. If $x = 5\,\text{m}$, what is the perimeter of the plot?

c. What shape is the plot of land?

11. **a.** Add $\left(4x^2+3x-2\right)$ and $\left(x^2-x+5\right)$.

b. Check your answer to question 11.a. by substituting 2 for x.

Check your answers by turning to the Appendix.

Subtracting Polynomials

You can also use algebra tiles to help with subtracting polynomials.

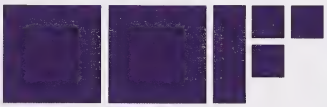
Watch the remainder of the video entitled *Tilly Tile* on subtraction of polynomials. Use algebra tiles to do the video assignment. This video is available from the Learning Resources Distributing Centre.

Check your answers to the video assignment by viewing the video.

Use the following Tile Diagrams to answer questions 12 to 15.

Tile Diagram A

Tile Diagram B



12. **a.** How many tiles of each kind (x^2-tiles, x-tiles, and 1-tiles) are in Tile Diagram A?

b. How many tiles of each kind are in Tile Diagram B?

13. If you subtract the tiles in Tile Diagram B from the tiles in Tile Diagram A, how many tiles of each type are left?

14. Write a polynomial that represents the following:

a. Tile Diagram A

b. Tile Diagram B

c. Tile Diagram A minus Tile Diagram B

15. Change all the tiles for Tile Diagram B to coloured tiles to make its opposite.

a. Write the polynomial that expresses the sum of Tile Diagram A and the new Tile Diagram B.

b. Compare the answer to question 15.a. to the answer to question 14.c. What do you notice?

c. What does the answer to question 15.b. suggest about subtracting polynomials?

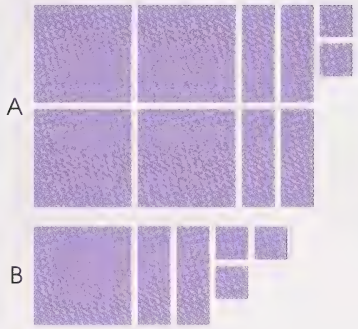
Check your answers by turning to the Appendix.

You can subtract one polynomial from another by adding its opposite. That is, take the opposite of each term of the polynomial that you are subtracting and add it to the corresponding like term of the other polynomial.

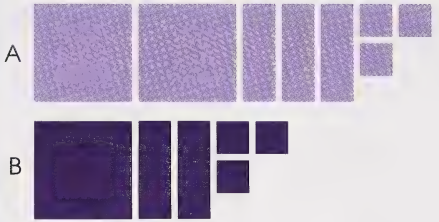
16. Find the difference of the algebra tiles in each pair of tile diagrams; then write the equivalent subtraction using polynomials.

a.

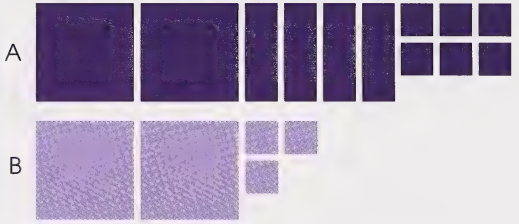
b.



c.



d.



17. After completing question 16, explain how polynomials can be subtracted without using algebra tiles.

Check your answers by turning to the Appendix.

The following examples show how to find the difference between two polynomials using modelling and algebraic methods.

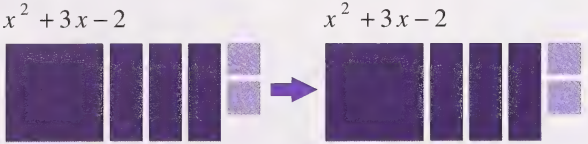
Example 1

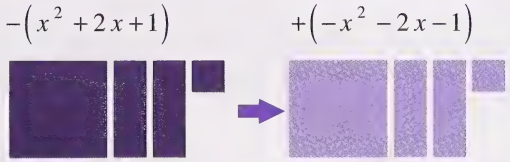
Find the difference of $\left(x^2+3x-2\right)$ and $\left(x^2+2x+1\right)$.

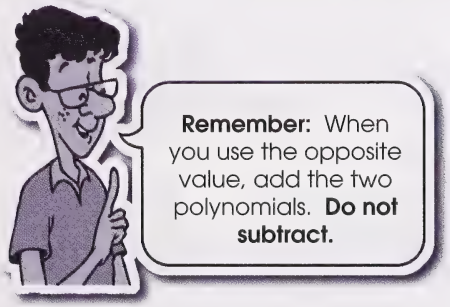
Solution

Method 1

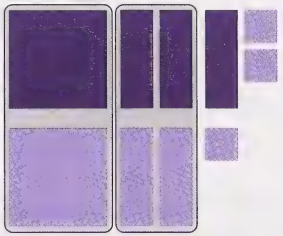
Step 1: Model each polynomial using the opposite value of each tile in the polynomial you are subtracting.



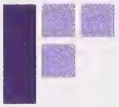




Step 2: Use the zero principal to simplify.



Step 3: Combine like terms.



Verification

You can verify by picking a value for x and substituting that value in the expression. Try $x=2$.

LS	RS
$\left(x^2+3x-2\right)-\left(x^2+2x+1\right)$	$x-3$
$=\left[(2)^2+3(2)-2\right]-\left[(2)^2+2(2)+1\right]$	$=2-3$
$=(4+6-2)-(4+4+1)$	$=-1$
$=(8)-(9)$	
$=-1$	

LS = RS

Therefore, $\left(x^2+3x-2\right)-\left(x^2+2x+1\right)=x-3$.

Can you see how the same polynomials are subtracted without using algebra tiles? You can do the subtraction in one line or by lining up the terms vertically in column form.

Method 2

Change each term in the polynomial you are subtracting to its opposite, and change the operation to addition. Then remove the parentheses, collect the like terms, and simplify.

$$\begin{aligned}\left(x^2+3x-2\right)-\left(x^2+2x+1\right) &= x^2+3x-2+\left(-x^2-2x-1\right)\\ &= x^2+3x-2-x^2-2x-1\\ &= x^2-x^2+3x-2x-2-1\\ &= x-3\end{aligned}$$

Method 3

Use column form to subtract the polynomials.

$$\begin{array}{r} x^2+3x-2 \\ -\left(x^2+2x+1\right) \\ \hline \end{array} \quad\Rightarrow\quad \begin{array}{r} x^2+3x-2 \\ +\left(-x^2-2x-1\right) \\ \hline 0x^2+x-3 \end{array}$$

$1+(-1)$ $\quad 3+(-2)$ $\quad -2+(-1)$

Verification

The verification is the same as the one that was done in Method 1.

Therefore, $\left(x^2+3x-2\right)-\left(x^2+2x+1\right)=x-3$.

Example 2

Determine the difference of $\left(-3x^2+2x-1\right)$ and $\left(2x^2+3x\right)$.

Solution

Method 1

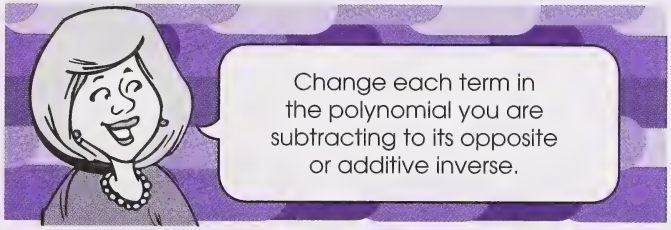
Change each term in the polynomial you are subtracting to its opposite; then remove the parentheses, collect like terms, and simplify.

$$\begin{aligned}\left(-3x^2+2x-1\right)-\left(2x^2+3x\right) &= \left(-3x^2+2x-1\right)+\left(-2x^2-3x\right)\\ &= -3x^2+2x-1-2x^2-3x\\ &= -3x^2-2x^2+2x-3x-1\\ &= -5x^2-x-1\end{aligned}$$

Method 2

Use column form to subtract the polynomials.

$$\begin{array}{r} -3x^2+2x-1 \\ -\left(2x^2+3x\ \ \right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} -3x^2+2x-1 \\ +\left(-2x^2-3x\ \ \right) \\ \hline -5x^2-\ \ x-1 \end{array}$$



Therefore, $\left(-3x^2+2x-1\right)-\left(2x^2+3x\right)=-5x^2-x-1$.

18. Find the difference of the following polynomials using algebra tiles. Remember to change those tiles for the polynomial being subtracted to its opposite. Show the equivalent algebraic solution for each.

a. $\left(2x^2+3x+4\right)-\left(x^2+x+4\right)$

b. $\left(x^2-x-4\right)-\left(2x^2+x-1\right)$

c. $\left(-2x^2+x-2\right)-\left(x^2+3x+4\right)$

d. $\left(x^2+2\right)-\left(-2x^2+4x\right)$

19. Subtract the following polynomials. Take the opposite of each term in the polynomial being subtracted; then remove the parentheses, collect like terms, and simplify.

a. $\left(4y^2+3y+5\right)-\left(y^2+3y+2\right)$

b. $\left(-3y^2-y+7\right)-\left(2y^2-3y+3\right)$

c. $\left(3y^3+3y^2-2y+5\right)-\left(y^3-2y^2+4y-2\right)$

d. $\left(y^2-4y+5\right)-\left(y^2+6\right)$

20. Subtract the following using the column method.

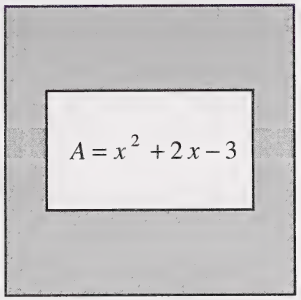
a.
$$\begin{array}{r} 4a^2+3a+5 \\ -\left(2a^2+2a+3\right) \\ \hline \end{array}$$

b.
$$\begin{array}{r} 2b^2+7b-3 \\ -\left(\ b^2-2b-1\right) \\ \hline \end{array}$$

c.
$$\begin{array}{r} -6c^2+5c-8 \\ -\left(4c^2\ \ \ \ +7\right) \\ \hline \end{array}$$

d.
$$\begin{array}{r} 8n^3+4n^2-2n+7 \\ -\left(2n^3-3n^2+\ n+2\right) \\ \hline \end{array}$$

21. **a.** Write an expression for the area of the shaded region in the diagram if the area of the large square is $4x^2 + 4x + 1$.



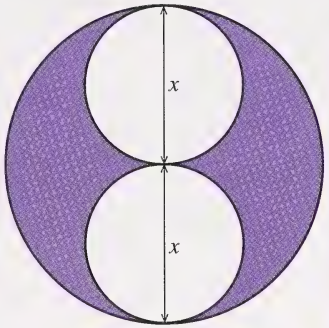
b. Write the expression in question 21.a. in simplest form.

c. If $x = 4$ cm, what is the area of the shaded region?

22. Find the puzzle "Daffynition Decoder" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle. You may use any method to subtract the polynomials.

23. **a.** Write an expression that represents the area of the shaded region. (Area of a circle is πr^2.)

b. How does the area of the coloured region compare to the area of the two small circles?



Use a calculator to answer question 24.

24. Mary makes clay pottery. The temperature T of her drying oven (in °C), m minutes after it has been turned on, is given by the following formula.

$$T = -m^2 + 28m + 35$$

After 12 min, the temperature stays constant.

a. Find the present temperature of the oven if it was turned on four minutes ago.

b. Find the present temperature of the oven if it was turned on nine minutes ago.

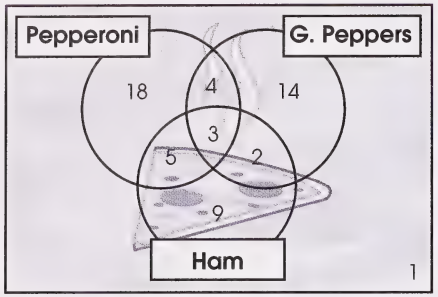
c. Determine the maximum temperature at which the oven operates.

Check your answers by turning to the Appendix.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

25. The students in classes 9A and 9B at Middle Junior High School were asked whether or not they like pepperoni, green peppers, and/or ham on their pizza. The results of the 56 students surveyed are shown in the following Venn diagram.



a. How many students liked each of the following toppings?

i. pepperoni
ii. pepperoni only
iii. green peppers
iv. green peppers only
v. ham
vi. ham only
vii. pepperoni and green peppers together
viii. pepperoni and ham together
ix. green peppers and ham together
x. all three toppings
xi. green peppers and pepperoni, but not ham

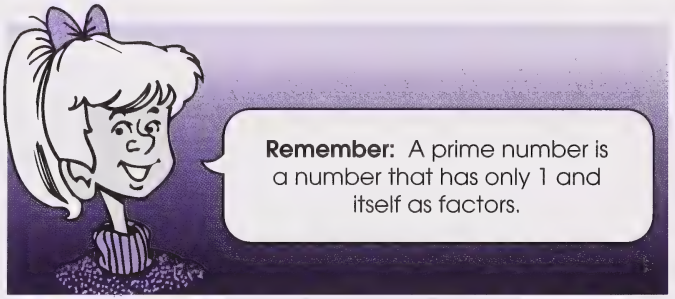
b. How many students did not like any of these toppings?

Check your answers by turning to the Appendix.

Searching for Prime Numbers

If you have access to a computer spreadsheet program, try the following exercise.

Polynomials can be used in a spreadsheet to generate prime numbers. The polynomial $n^2 - n + 41$ for the generation of primes was first developed by Leonard Euler, a Swiss mathematician, in 1772.



The polynomial $n^2 - 79n + 1601$ was developed by E.B. Escott in 1879.

Set up a spreadsheet similar to the following and test both Euler's polynomial, and Escott's polynomial for several values of n.

	A	B	C
1	Number	Euler's Polynomial	Escott's Polynomial
2		=A2*A2-A2+41	=A2*A2-79*A2+1601
3			
4			
5			

You can check if the numbers you get in cells B2 or C2 are prime using a calculator.

To test for a prime.

- Divide the number by all prime numbers between 1 and the square root of the number you are testing.
- If you do not get any whole-number quotients, then the number is prime.

If you have access to the Internet you can check if the number is prime by finding it in the list of the first 10 000 primes given at the following URL.

http://www.utm.edu:80/research/primes/notes/10000.txt

You may stop the data transmission after you have the first page or two of primes.

26. Are the primes you obtain for Euler's polynomial the same as those you obtain for Escott's polynomial?

27. Can you find any values of n for which the same value is produced by both polynomials?

28. Is this result a prime?

29. Try values of n above and below $n = 20$. What patterns for the values from Euler's and Escott's polynomials do you see?

30. Try $n = 41$. Is the value obtained from each polynomial a prime?

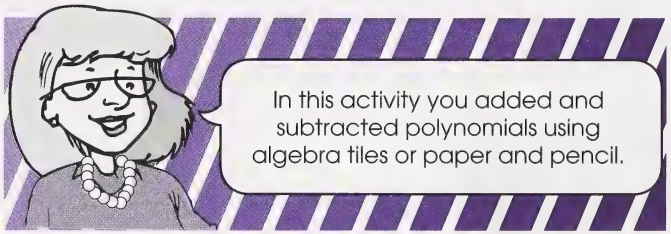
31. Try $n = 80$. Is the value obtained from Escott's polynomial a prime number?

32. Hypothesize the values of n that will provide a prime number for each polynomial.

33. Are the values for Euler's polynomial for $n > 41$ prime numbers?

34. What conclusion can you reach regarding polynomials developed to produce prime numbers.

Check your answers by turning to the Appendix.



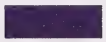
Activity 3: Multiplying Monomials

To find the area of a rectangle, multiply the length by the width. For instance, to find the area of the photograph of the Eiffel Tower in Paris, France, multiply 5 cm by 4 cm.

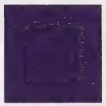
$$A = 4 \times 5$$
$$= 20 \text{ cm}^2$$

You can apply the concept of area of a rectangle to multiplying monomials by forming a rectangle with the algebra tiles that represent the product of the two monomials.

The 1-tile is 1 unit by 1 unit; therefore, it has an area of $1 \times 1 = 1$ square unit.

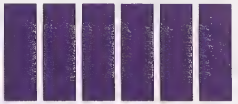


The x-tile is 1 unit by x units; therefore, it has an area of $1 \times x = x$ square units.



The x^2-tile is x units by x units; therefore, it has an area of $x \times x = x^2$ square units.

1. Use the algebra tiles on the right to answer the following questions.

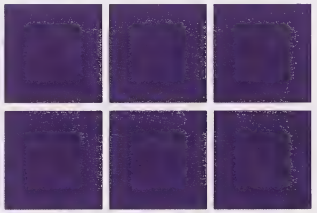


a. What is the area of the following rectangular group of tiles?

b. What is the length of the rectangle? What is the width of the rectangle?

c. What is the area, expressed as a product of its dimensions?

2. Use the algebra tiles on the right to answer the following questions.



a. What is the area of the rectangular group of tiles?

b. What is the length of the rectangle? What is the width of the rectangle?

c. What is the area as a product of its dimensions?

Check your answers by turning to the Appendix.

To illustrate the multiplication of monomials, you can set up a partial rectangle (as shown in Example 1). You can show what monomials are being multiplied by placing them across the top and down the left side.

Example 1

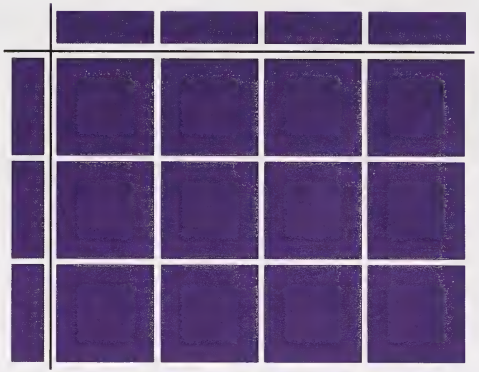
Find the product of $4x$ and $3x$.

Solution

Use tiles to represent the monomials and the product.

Step 1: Lay out four x-tiles horizontally and three x-tiles vertically.

Step 2: Fill in the rectangle with x^2-tiles, since x times x is x^2.



Step 3: Count the numbers of x^2-tiles. The number of x^2-tiles is the area and represents the product of $4x$ and $3x$.

There are 12 x^2-tiles. Therefore, the area is $12x^2$ and the product of $4x$ and $3x$ is $12x^2$.

Algebraically, the solution to Example 1 is as follows:

$$(4x)(3x) = (4)(3)(x)(x)$$
$$= 12x^2$$

3. Using algebra tiles, form rectangles with the given dimensions. Find the area of each rectangle, and state the area using algebra.

a. length = 4 and width = x

b. length = 3 and width = $2x$

c. length $= 2x$ and width $= 5$

Check your answers by turning to the Appendix.

Now study another example using algebra.

Example 2

Find the product of $6x$ and $4x$.

Solution

$$(6x)(4x) = (6)(4)(x)(x)$$
$$= 24x^2$$

4. Study the algebraic calculations in Examples 1 and 2. Describe a method you could use to find the product of two monomials without using algebra tiles.

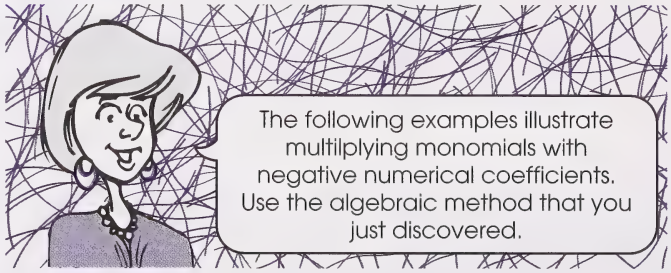
5. Determine the following products algebraically.

a. $(2x)(4x)$ **b.** $(3x)(5x)$ **c.** $(5)(2x)$
d. $(x)(4)$ **e.** $(3)(6x)$ **f.** $(2x)(3x)$

Check your answers by turning to the Appendix.

The following key idea sums up how you multiply two or more monomials.

To multiply two or more monomials, multiply their numerical coefficients and multiply their variables.



Example 3

Find the product of $-3x$ and $-4x$.

Solution

Write the two monomials as a product.

$$(-3x)(-4x)$$

Multiply the numerical coefficients, and multiply the variables.

$$(-3)(-4)(x)(x) = 12x^2$$

Remember: The product of two negative numbers is a positive number.

Example 4

Find the product of $(-y^2)(y)(3y^3)$.

Solution

Multiply the coefficients, and multiply the variables.

$$(-y^2)(y)(3y^3)$$
$$= -1(3)(y^2)(y)(y^3)$$
$$= -3y^6$$

Remember: Add exponents when multiplying numbers or variables with the same base.

6. Find the products of the following monomials.

a. $(2m)(2n)$

b. $(3a)(4a^3)$

c. $(6x)(3y^2)$

d. $(2x^2)(-4xy^2)$

e. $(-5ab)(-2ab)$

f. $(4x)(3xyz)$

g. $(-4a)(2b^2)$

h. $(3x^2y^2)(4y^3)$

i. $2ab \times 3ac$

j. $3a^2b^3c \times 4ab^2c$

k. $(2m)(3m)(4m^2)$

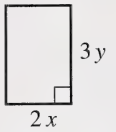
l. $(-2a^2)(3ab)(a^3b^2)$

m. $(2a)(2a)(2a)$

n. $(2a)^3$

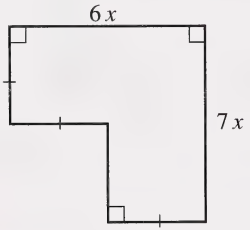
7. Write an expression for the area of each figure; then simplify.

a.

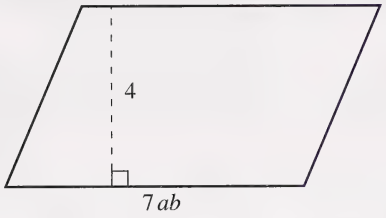


b.

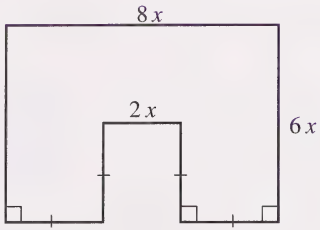
c.



d.

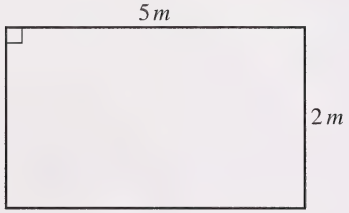


e.

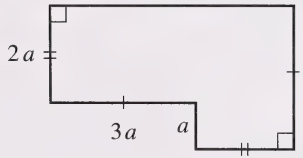


8. Find the perimeter and area of the following:

a.



b.

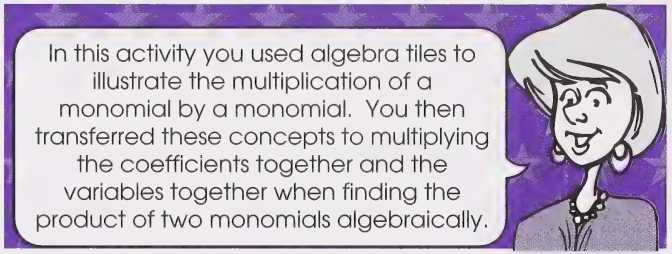


9. Express $6ab$ as a product of two monomials in as many different ways as you can. Use whole numbers only for your numerical coefficients. Consider different groupings as different ways.

10. A product like $2x^2$ and $3x$ cannot be illustrated using algebra tiles. Think of a type of algebra tile that you could make that would allow you to illustrate the product of $2x^2$ and $3x$.

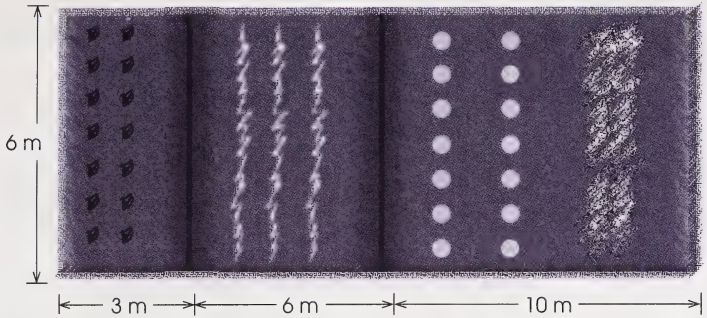
11. Henrietta wants to model $(2x)(3y)$ using algebra tiles. Show one way she can model these monomials and explain why you chose these particular tiles.

Check your answers by turning to the Appendix.



Activity 4: Multiplying a Polynomial by a Monomial

The Muttarts have divided their garden into three different plots (as shown in the following diagram). Can you find the total area of their garden in two different ways?

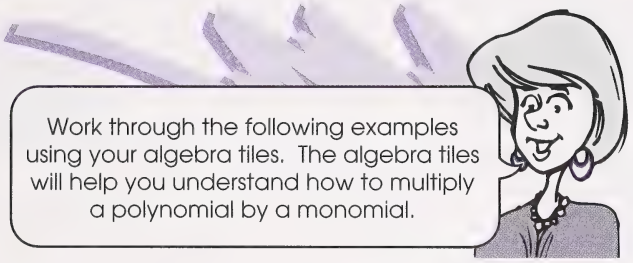


$6\times3+6\times6+6\times10$ or $6(3+6+10)$

The Muttarts' garden has an area of 114 m^2.

The same principle illustrated here (and used in the previous module to solve some equations) can be used to multiply a polynomial by a monomial.

Multiplying a polynomial by a monomial can be similar to finding the area of the Muttarts' garden.



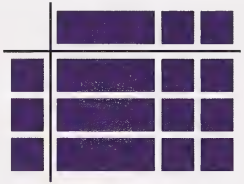
Example 1

Find the product of $3(x+2)$. Use algebra tiles to illustrate the process.

Solution

Step 1: Illustrate both 3 and $x+2$.

Step 2: Fill in the rectangle formed by the two sets of tiles.



Step 3: Read the product from the area of the rectangle under and to the right of the lines.

The area is $3x+6$.

Therefore, the product of 3 and $(x+2)$ is $3x+6$.

Algebraically, the solution to Example 1 is as follows:

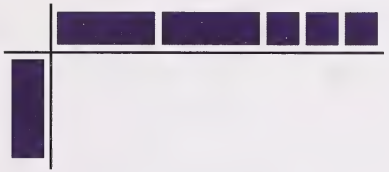
$$3(x+2) = 3(x) + 3(2)$$
$$= 3x + 6$$

Example 2

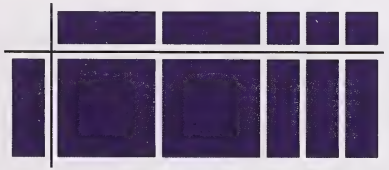
Find the product of x and $(2x+3)$. Use algebra tiles to illustrate the process.

Solution

Step 1: Illustrate both x and $2x+3$.



Step 2: Fill in the rectangle.



Step 3: Read the product from the area of the rectangle under and to the right of the lines.

The area is $2x^2 + 3x$.

Therefore, the product of x and $(2x+3)$ is $2x^2 + 3x$.

Algebraically, the solution to Example 2 is as follows:

$$x(2x+3) = x(2x) + x(3)$$
$$= 2x^2 + 3x$$

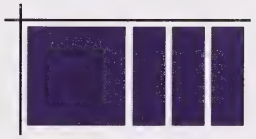
Examples 1 and 2 gave an algebraic method for multiplying a polynomial by a monomial.

1. What does the algebraic method suggest about how to multiply a polynomial by a monomial?

2. What is the expression that represents the area of each of these rectangles?

a.

b.



c.

d.

3. Use algebra tiles to show each dimension in question 2; then express each area as a product of its dimensions algebraically.

4. Find the product of each pair of dimensions in question 3. Are the answers here equivalent to those in question 2?

5. Find the area for each of the following using algebra tiles.

a. $4(x+2)$ **b.** $x(x+3)$

c. $x(2x+1)$ **d.** $2x(x+3)$

Check your answers by turning to the Appendix.

The following key idea sums up how you multiply a polynomial by a monomial.

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial. By doing this, you are using the **distributive property**.

The distributive property can be demonstrated using whole numbers.

LS	RS
$2(3+4+5)$	$2(3+4+5)$
$=2(12)$	$=2(3)+2(4)+2(5)$
$=24$	$=6+8+10$
	$=24$

LS = RS

Whether you multiply the number outside the brackets by each of the addends and add, or multiply the number outside the brackets by the sum, the result is the same.

Some polynomials will involve negative values. The next two examples illustrate how you can use algebra tiles to model multiplication of polynomials and monomials involving negative terms. The examples also illustrate how to use the distributive property to multiply polynomials and monomials with negative terms algebraically.

Example 3

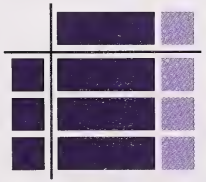
How do you find the product of $3(x-1)$?

Solution

Method 1

To show this product using algebra tiles, you need to form a rectangle with a width of 3 and a length of $x-1$.

Thus, the product of $3(x-1)$ would be represented by the completed rectangle.



Step 2 and Step 3 in the previous examples have been combined into one step here.

The area of the rectangle is $3x-3$. Therefore, $3(x-1)=3x-3$.

Method 2

Use the distributive property and multiply.

$$3(x-1)=3(x)+3(-1)$$
$$=3x-3$$

By removing the brackets and simplifying, you are **expanding** the expression.

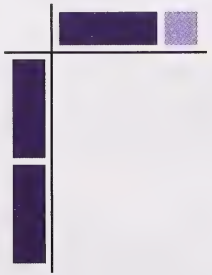
Example 4

Find the product of $2x(x-1)$.

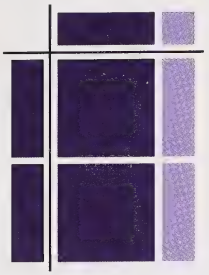
Solution

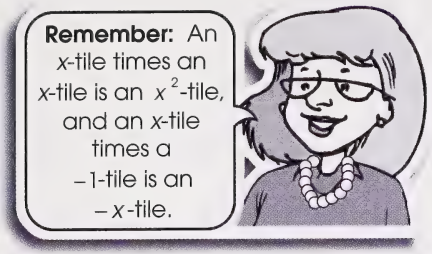
Method 1

Set up a rectangle with a width of $2x$ and a length of $x-1$.



Complete the rectangle using algebra tiles to show the product.





The area of the rectangle is $2x^2 - 2x$. Therefore,
$2x(x-1) = 2x^2 - 2x$.

Method 2

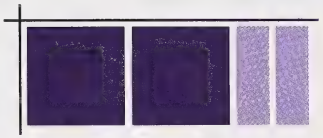
Use the distributive property and multiply.

$$2x(x-1) = 2x(x) + 2x(-1)$$
$$= 2x^2 - 2x$$

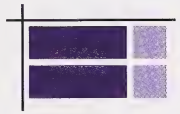
6. Use algebra tiles to show the dimensions of each rectangle. Then write an expression for the area of each rectangle as a product of its dimensions and as a single polynomial.

a.

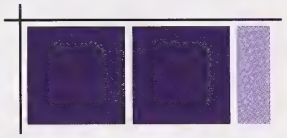
b.



c.



d.



7. Use algebra tiles to construct rectangles that represent these products, and find the areas.

a. $5(x-1)$ **b.** $x(3x-1)$

c. $x(x-2)$ **d.** $x(x-1)$

Check your answers by turning to the Appendix.

Polynomials that involve the product of x^2- and x-terms are not demonstrated using algebra tiles. However, by this point you should be familiar enough with using the distributive property to use the algebraic method only.

Example 6

Find the product of $3x\left(x^2+2x-4\right)$.

Solution

Use the distributive property.

$$3x\left(x^2+2x-4\right)=3x\left(x^2\right)+3x(2x)+3x(-4)$$
$$=3x^3+6x^2-12x$$

8. Determine the following products.

a. $4(x-1)$

b. $a(a+4)$

c. $5\left(x^2+x+1\right)$

d. $2b(b-3)$

e. $a(2a+3)$

f. $2y\left(y^2-y+3\right)$

9. Expand the following:

a. $2n\left(n^2+3\right)$

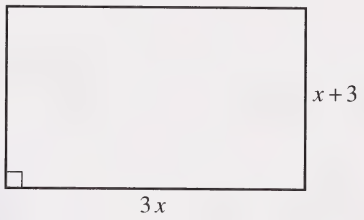
b. $3d\left(d^2+2d-4\right)$

c. $2\left(a^2+2a+3\right)+3\left(a^2-a+1\right)$

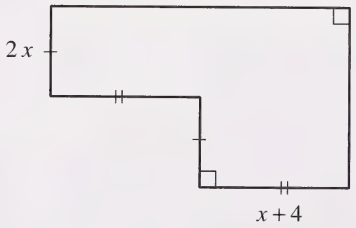
d. $-3a\left(a^2+a-4\right)$

10. Write an expression for the area of each figure. Show how you obtained your answer.

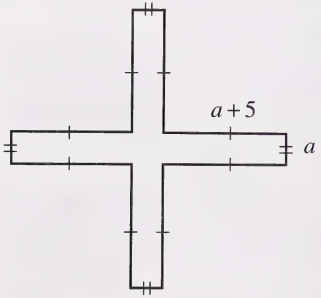
a.



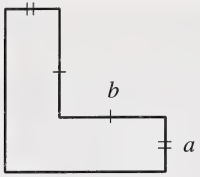
b.



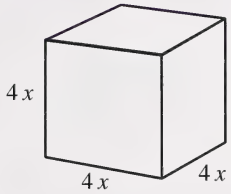
c.



11. Find the area of the following figure in two different ways.



12. Write the volume of the cube as a product of monomials; then express the answer as a single monomial.



13. If a stands for the smallest of two consecutive whole numbers, write an expression for their product.

14. Describe, in words, how you would find the product of two consecutive whole numbers using the expression from question 13.

15. Does the rule from question 14 also apply for consecutive negative integers? Explain with an example.

Check your answers by turning to the Appendix.

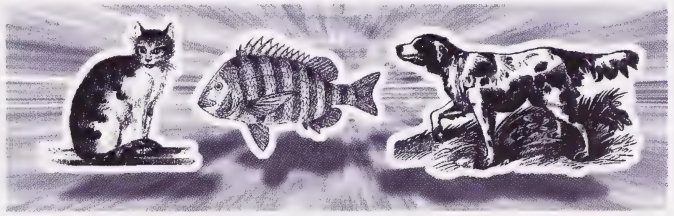
Now Try This

Use one of the problem-solving strategies to solve the following problem.

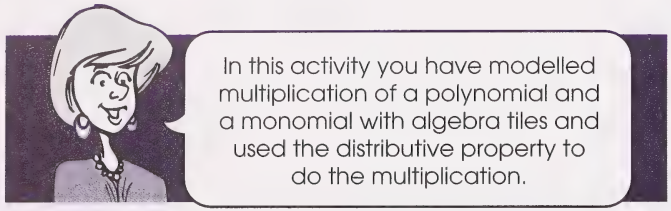
16. In a neighbourhood of 50 families, the following facts are known.

- 27 have dogs
- 25 have cats
- 18 have tropical fish
- 5 have cats and dogs
- 3 have dogs and tropical fish
- 2 have cats and tropical fish
- 8 have all three kinds of pets

How many families do not have cats, dogs, or fish?

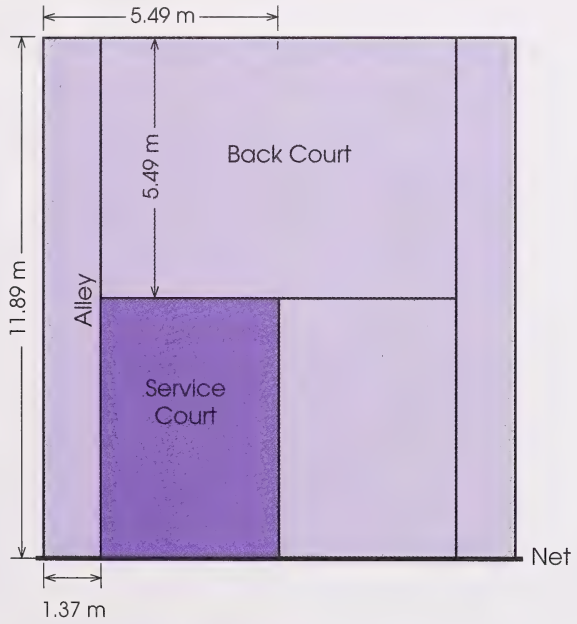


Check your answer by turning to the Appendix.



Activity 5: Multiplying Binomials

A tennis court is subdivided into several areas.



How big is the service court in tennis? To find the area of the service court, you must find the product of $(11.59-5.49)(5.49-1.37)$. The calculations used to find the answer could be compared to multiplying binomials.

Multiplying binomials is an extension of multiplying a polynomial by a monomial. You will see how as you study the examples and work through the activities that follow.

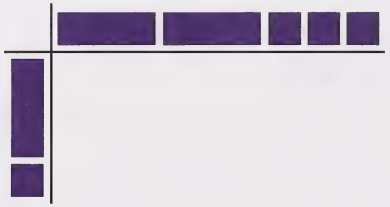
Use algebra tiles to follow Examples 1 and 2.

Example 1

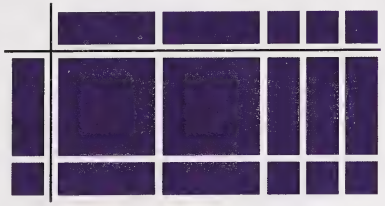
Find the product of $(x+1)$ and $(2x+3)$ using algebra tiles.

Solution

Step 1: Lay out two x-tiles and three 1-tiles horizontally and one x-tile and one 1-tile vertically.



Step 2: Fill in the rectangle.



Step 3: Count the tiles in the rectangle. This area represents the product of $(x+1)$ and $(2x+3)$.

The area is $2x^2+5x+3$.

Algebraically, the answer to Example 1 is as follows:

$$(x+1)(2x+3) = (x)(2x)+(x)(3)+(1)(2x)+(1)(3)$$
$$= 2x^2+3x+2x+3$$
$$= 2x^2+5x+3$$

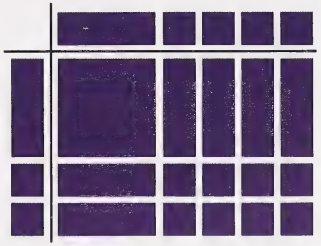
Example 2

Find the product of $(x+4)$ and $(x+2)$ using algebra tiles.

Solution

Step 1: Lay out one x-tile and four 1-tiles horizontally and one x-tile and two 1-tiles vertically.

Step 2: Fill in the rectangle.



Step 3: Count the tiles in the rectangle. This area represents the product of $(x+4)$ and $(x+2)$.

The area is x^2+6x+8.

Algebraically, the answer to Example 2 is as follows:

$$\begin{aligned}(x+4)(x+2) &= (x)(x)+(x)(2)+(4)(x)+(4)(2)\\ &= x^2+2x+4x+8\\ &= x^2+6x+8\end{aligned}$$

1. Use algebra tiles to illustrate each of the following products. Show the area (product) and give the product algebraically.

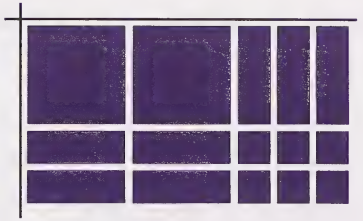
a. $(x+3)(x+1)$

b. $(2x+3)(x+1)$

2. Express the area of each rectangle as a product of its dimensions (length and width) using algebra tiles and algebraically.

a.

b.



3. Study the algebraic part of the solution in the Examples 1 and 2. Can you describe a method for finding the product of two binomials? Explain.

4. Use the method in question 3 to multiply these binomials.

a. $(x+1)(x+6)$ **b.** $(x+6)(x+3)$

c. $(x+4)(x+3)$ **d.** $(2x+3)(4x+2)$

Check your answers by turning to the Appendix.

To multiply a binomial by another binomial, multiply each term of the first binomial by each term of the second binomial; then collect the like terms.

You have analysed two methods of multiplying binomials in the previous two examples. The first method used algebra tiles while the second method used algebraic expressions. These same methods can be used when multiplying binomials involving negative terms.

Example 3

Find the product of $(2x+1)(x-1)$.

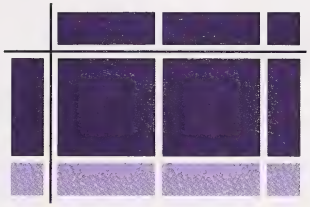
Solution

Method 1

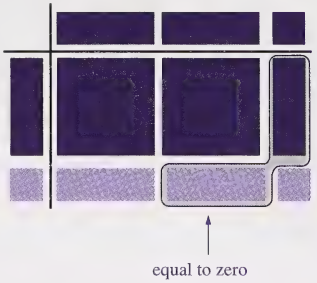
Use algebra tiles to find the product.

Step 1: Lay out two x-tiles and one 1-tile horizontally and one x-tile and one -1-tile vertically.

Step 2: Fill in the rectangle.



Step 3: Use the zero principle to simplify.



Step 4: Count the remaining tiles in the rectangle. This represents the product of $(2x+1)$ and $(x-1)$.

The product is $2x^2 - x - 1$.

Method 2

Multiply each term of the first binomial by each term of the second binomial. The answer is as follows:

$$\begin{aligned}(2x+1)(x-1) &= (2x)(x)+(2x)(-1)+(1)(x)+(1)(-1)\\ &= 2x^2 - 2x + x - 1\\ &= 2x^2 - x - 1\end{aligned}$$

Example 4

Find the product of $(3x-2)(2x-1)$.

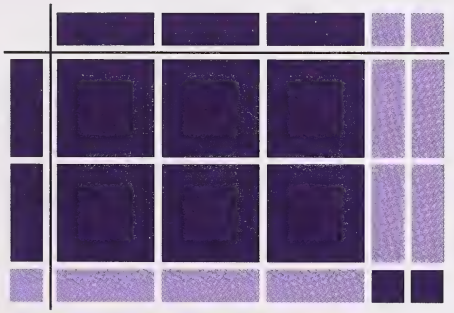
Solution

Method 1

Use algebra tiles to find the product.

Step 1: Lay out three x-tiles and two -1-tiles horizontally and two x-tiles and one -1-tile vertically.

Step 2: Fill in the rectangle.



Step 3: There are no tiles that zero out. Thus, count the tiles in the rectangle.

The area is $6x^2 - 7x + 2$.

Therefore, $(3x-2)(2x-1) = 6x^2 - 7x + 2$.

Method 2

Multiply each term of the first binomial by each term of the second binomial.

$$\begin{aligned}&(3x-2)(2x-1)\\ &\quad = (3x)(2x) + (3x)(-1) + (-2)(2x) + (-2)(-1)\\ &\quad = 6x^2 - 3x - 4x + 2\\ &\quad = 6x^2 - 7x + 2\end{aligned}$$

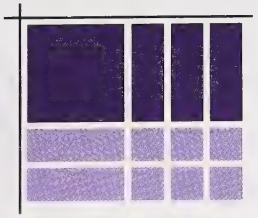
5. Use algebra tiles to make rectangles that could be used to illustrate each of the following products. Count the tiles and state the product (area) of each.

a. $(x+2)(x-1)$

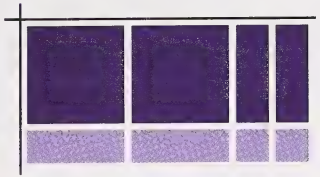
b. $(x+1)(x-2)$

6. Express the area of each rectangle as a product of its length and width. Find the area by multiplying the binomials algebraically. Verify by counting the tiles in the rectangle.

a.



b.



7. Find the product of each of the following. Use algebra tiles if necessary.

a. $(x+4)(x+3)$ **b.** $(x-2)(x+5)$

c. $(2x-3)(x+4)$ **d.** $(x+6)(x-1)$

e. $(2x+6)(x+2)$ **f.** $(x-1)(x-1)$

8. a. What does n^2 mean?

b. Expand $(x+2)^2$.

c. Find the product of $(x+3)^2$.

d. The answers to 8.b and 8.c are called perfect square trinomials. Is $x^2+8x+16$ a perfect square trinomial? Explain.

9. Determine the following products.

a. $(x-4)(x+4)$ **b.** $(a+3)(a-3)$

c. $(2x-1)(2x+1)$ **d.** $(y-2)(y+2)$

10. a. What are the similarities in each of your answers in question 9?

b. What's special about the two terms in each product?

c. Products like these are called differences of perfect squares. They come from multiplying two binomials that differ only in the sign on the second term. Would the product $(x-5)(x+5)$ give you a difference of perfect squares? Explain.

d. Use algebra tiles to illustrate the multiplication of $(x-5)$ and $(x+5)$. How do the algebra tiles help you to see the resulting product?

11. Determine the following products.

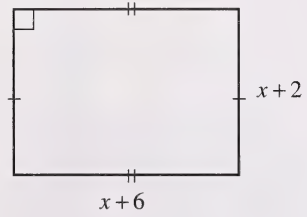
a. $(2x+3)(2x-3)$ **b.** $(x+4)(x-1)$

c. $(x+3)(x-5)$ **d.** $(x+7)^2$

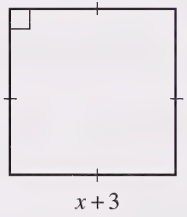
e. $(2x+5)(x+2)$ **f.** $(x-9)(x-6)$

12. Write an expression for the perimeter and the area of each figure; then expand each expression to find the perimeter and area.

a.



b.



13. Find the puzzle "Why Is a Stick of Gum Like a Sneeze?" in the Appendix. Either photocopy the puzzle or pull out the page. Then complete the puzzle.

Check your answers by turning to the Appendix.

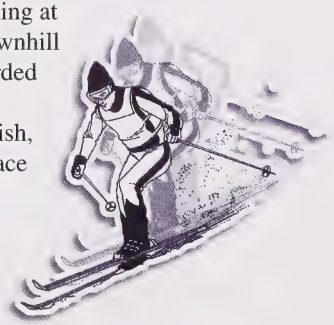
Now Try This

Solve the following challenge problem.

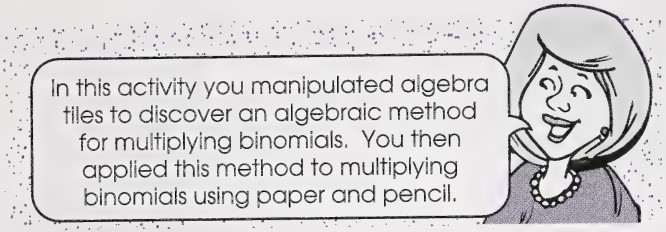
14. Which two consecutive whole numbers have squares that differ by 81? Solve this problem in two different ways, one of which must be algebraically (that is, using an equation).

Use one of the problem-solving strategies to solve the following problem.

15. To determine their overall standing at the end of the season, all the downhill skiers who competed were awarded five points for each win, three points for each second place finish, and two points for each third place finish. If Marten accumulated 15 points, what possible combinations of first, second, and third places could he have achieved over the season?

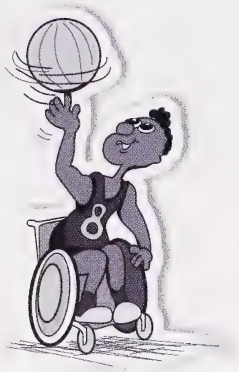


Check your answers by turning to the Appendix.



Activity 6: Dividing by a Monomial

Have you ever played three-on-three basketball or touch football? When you get together with your friends for a game of three-on-three basketball or touch football, you put yourselves into groups. Division is deciding how many groups of the divisor there are in the dividend.



Dividing by a Constant

You have been performing the operation of division with whole numbers for several years.

1. What does it mean when you divide 12 by 2?

Check your answer by turning to the Appendix.

$12 = 2+2+2+2+2+2$

There are six groups of 2 in 12.

Therefore, $12 \div 2 = 6$.

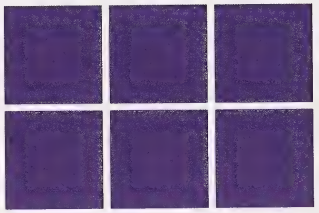
You follow the same procedure when dividing a polynomial by a constant.

Example 1

Divide $6x^2$ by 3.

Solution

Use tiles to represent $6x^2$.



Divide the tiles into three equal groups. How many x^2-tiles are there in each group?

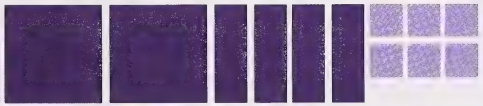
There are two x^2-tiles in each group. Therefore, $6x^2 \div 3 = 2x^2$.

Example 2

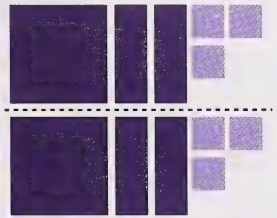
Divide $\left(2x^2 + 4x - 6\right)$ by 2.

Solution

Use tiles to represent $2x^2 + 4x - 6$.



Divide the tiles into two equal groups.



Count the number of tiles of each type there are in each group.

There are one x^2-tile, two x-tiles, and three -1-tiles in each group.

Therefore, $\left(2x^2 + 4x - 6\right) \div 2 = x^2 + 2x - 3$.

2. Use algebra tiles to perform each of the following divisions.

a. $8x^2 \div 2$

b. $6x \div 3$

c. $\left(2x^2 + 4\right) \div 2$

d. $\left(-2x^2 - 6x + 4\right) \div 2$

e. $(9x + 6) \div 3$

f. $\left(4x^2 + 12x - 8\right) \div 4$

3. Describe a rule that you could use to divide a polynomial by a constant without using algebra tiles.

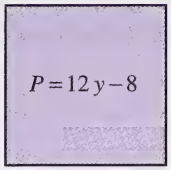
4. Use the rule from question 3 to perform the following divisions.

a. $(6x - 9) \div 3$

b. $\left(4x^2 + 2x - 2\right) \div 2$

c. $\dfrac{6x^2 + 9x - 3}{3}$

5. The perimeter of the square on the right is given.



a. Find the length of a side.

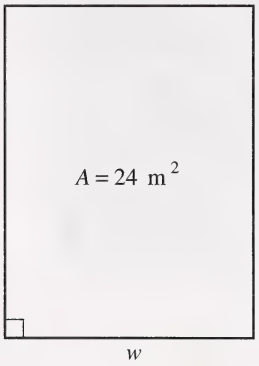
b. Find the area.

Check your answers by turning to the Appendix.

Dividing by a Variable

If you were asked to divide $\left(4x^2 + 2x\right)$ by $2x$, you would be unable to use the grouping model that was used when dividing by monomials. But, since division is the opposite of multiplication ($8 \div 4 = 2$ and $2 \times 4 = 8$), you can use the area model used for multiplying.

If the area of this rectangle is $24\ \text{m}^2$ and its length is 6 m, what is its width?



Since $A = \ell \times w$, then $w = A \div \ell$.

$\therefore\ w = 24 \div 6$

$= 4\ \text{m}$

$6 \times 4 = 24$

The width of the rectangle is 4 m.

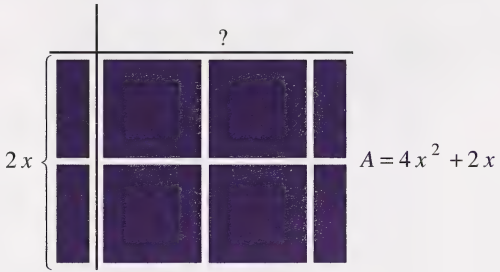
You determined the area of the rectangle by dividing the area by the known dimension. Similarly, you can divide a polynomial by a monomial containing a variable. Examples 1 and 2 illustrate this process using algebra tiles. This is similar to using algebra tiles to find the dimensions given the area. (This was done in Activity 4.)

Example 1

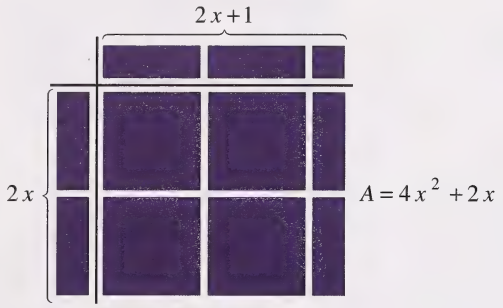
Divide $\left(4x^2 + 2x\right)$ by $2x$.

Solution

To divide $\left(4x^2 + 2x\right)$ by $2x$ using algebra tiles, form a rectangle with an area of $4x^2 + 2x$ and a width of $2x$.



Now lay out the tiles above the line to find the length of the rectangle.



The length of the rectangle is $2x+1$.

Therefore, $\left(4x^2+2x\right)\div 2x = 2x+1$.

Verification

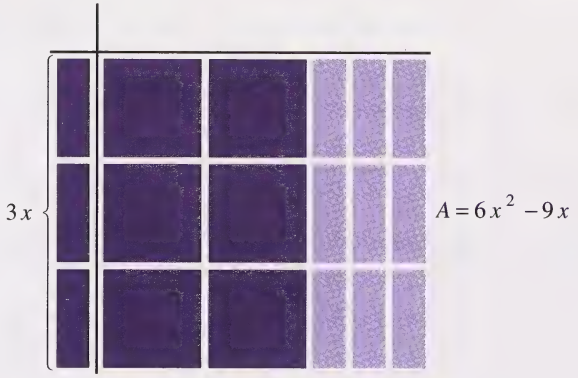
$$2x(2x+1) = 2x(2x)+2x(1)$$
$$= 4x^2+2x$$

Example 2

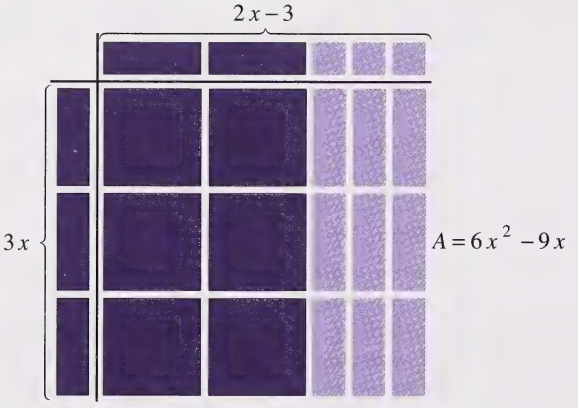
Divide $\left(6x^2-9\right)$ by $3x$.

Solution

Form a rectangle with an area of $6x^2-9$ and a width of $3x$.



Now lay out the tiles to find the length of the rectangle.



The length of the rectangle is $2x-3$.

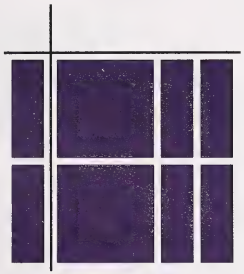
Therefore, $\left(6x^2-9\right)\div 3x = 2x-3$.

Verification

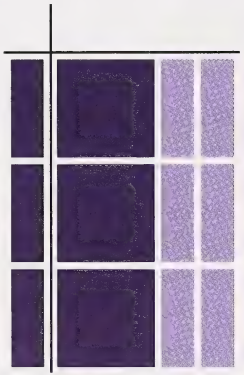
$$3x(2x-3) = 3x(2x)+3x(-3)$$
$$= 6x^2-9x$$

6. Use the rectangular display of algebra tiles to do the indicated division.

a. $\left(2x^2+4x\right)\div 2x$



b. $\left(3x^2-6x\right)\div 3x$



c. $\left(2x^2+3x\right)\div x$

d. $\left(6x^2-2x\right)\div 2x$

7. Use algebra tiles to perform the following divisions.

a. $\left(6x^2-4x\right)\div 2x$

b. $\left(6x^2+3x\right)\div 3x$

c. $\left(2x^2+6x\right)\div 2x$

d. $\left(3x^2-6x\right)\div 3x$

8. Describe a rule that you could use to divide a polynomial by a monomial that includes a variable without using the algebraic tiles.

Check your answers by turning to the Appendix.

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial. That is, divide the numerical coefficients and the literal coefficients each time.

Study the next two examples to see how the division of a polynomial by a monomial can be performed algebraically.

Example 3

Divide $\left(8x^2+6x\right)$ by $2x$.

Solution

$$\left(8x^2+6x\right)\div 2x=\frac{8x^2}{2x}+\frac{6x}{2x}$$

$$=4x+3$$

$\frac{8}{2}=4$

$\frac{x^2}{x}=x^{2-1}=x$

$\frac{6}{2}=3$ and $\frac{x}{x}=1$

Verification

$$2x(4x+3)=2x(4x)+2x(3)$$

$$=8x^2+6x$$

Example 4

Divide $\left(12x^3+8x^2-4x\right)$ by $4x$.

Solution

$$\frac{12x^3+8x^2-4x}{4x}=\frac{12x^3}{4x}+\frac{8x^2}{4x}+\frac{-4x}{4x}$$

$$=3x^2+2x-1$$

Remember: A negative term divided by a positive term gives a negative term.

Verification

$$4x\left(3x^2+2x-1\right)=4x\left(3x^2\right)+4x(2x)+4x(-1)$$

$$=12x^3+8x^2-4x$$

9. Perform the following divisions algebraically. Use the algebra tiles if necessary.

a. $\left(6x^2+5x\right)\div x$

b. $\left(4x^2+6x\right)\div 2x$

c. $\left(10x^2+5x\right)\div 5x$

10. Perform the following divisions algebraically. Verify your answers by multiplying.

a. $\dfrac{4x^2+2x+6}{2}$

b. $\dfrac{3x^2-6x}{3x}$

c. $\dfrac{5x^3+10x^2-15x}{5x}$

d. $\dfrac{x^2+4x}{x}$

Check your answers by turning to the Appendix.

In question 10, you verified your answers by multiplying. You can also verify your answer by substituting a number for the variable in the original expression and in the resulting quotient.

Example 5

Divide $\left(8x^3+4x^2-4x\right)$ by $-4x$.

Solution

$$\frac{8x^3+4x^2-4x}{-4x}=\frac{8x^3}{-4x}+\frac{4x^2}{-4x}+\frac{-4x}{-4x}$$

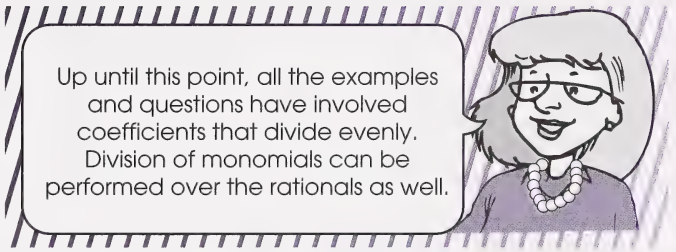
$$=-2x^2-x+1$$

Verification

Verify by substituting $x=1$.

LS	RS
$\dfrac{8(1)^3+4(1)^2-4(1)}{-4(1)}$	$-2(1)^2-(1)+1$
$=\dfrac{8+4-4}{-4}$	$=-2-1+1$
$=\dfrac{8}{-4}$	$=-2$
$=-2$	

LS = RS



Example 6

Divide $\left(3m^3+8m^2-5m\right)$ by $4m$.

Solution

$$\frac{3m^3+8m^2-5m}{4m}=\frac{3m^3}{4m}+\frac{8m^2}{4m}+\frac{-5m}{4m}$$

$$=\frac{3m^2}{4}+2m+\frac{-5}{4}$$

$$=\frac{3}{4}m^2+2m-\frac{5}{4}$$

Verification

Verify by substituting $m=1$.

LS	RS
$\frac{3(1)^3+8(1)^2-5(1)}{4(1)}$	$\frac{3}{4}(1)^2+2(1)-\frac{5}{4}$
$=\frac{3+8-5}{4}$	$=\frac{3}{4}+2-\frac{5}{4}$
$=\frac{6}{4}$	$=\frac{3}{4}+\frac{8}{4}-\frac{5}{4}$
$=\frac{3}{2}$ or $1\frac{1}{2}$	$=\frac{3+8-5}{4}$
	$=\frac{6}{4}$
	$=\frac{3}{2}$ or $1\frac{1}{2}$

LS = RS

11. Perform the following divisions. Verify your answer by substituting $x=1$.

a. $\left(4x^3+2x^2-6x\right)\div 2x$

b. $(6x-12)\div(-3)$

c. $\frac{4x^2+6x-8}{5}$

d. $\frac{8x^3+12x^2-4x}{-4x}$

12. Perform the following divisions.

a. $\left(6x^3+4x^2+8x\right)\div 3x$

b. $\left(12x^4+8x^3-16x^2+4x\right)\div(-4x)$

c. $\left(x^3-9x\right)\div x$

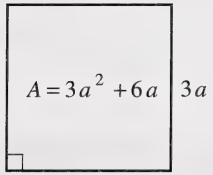
d. $\frac{-12y^3+9y^2-y}{-3y}$

e. $\frac{24-8m+4m^2}{4}$

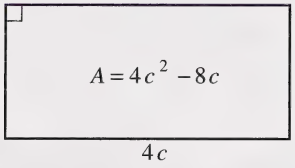
f. $\frac{14a^2b^3+7a^2b^2-7ab^2}{7ab}$

13. Find the width of each rectangle.

a.



b.



14. If the width of a rectangle is $2b$ and its area is $6b^2 + 2b$, find the following:

a. the length **b.** the perimeter

15. Find the value of each expression if $x = 2$ and $y = -3$.

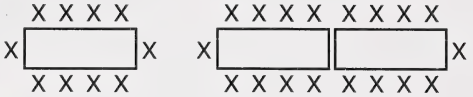
a. $\dfrac{6x^2y^2}{2xy}$ **b.** $3xy$

16. What do you notice about your answers in question 15? Explain.

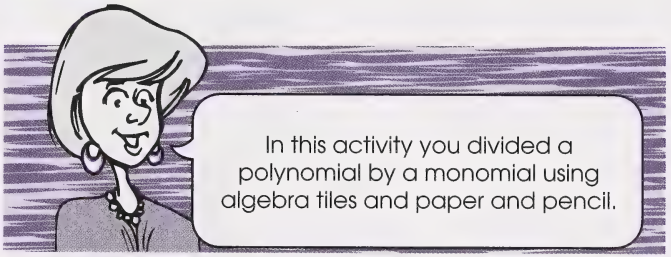
Now Try This

Use one of the problem-solving strategies to solve the following problem.

17. A banquet table can seat four people on each side and one person on each end. If 13 of these tables are put together in one long row, what is the maximum number of people that can be seated?



Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may do both.

Extra Help

When you use algebra tiles to subtract polynomials, you do not need to represent the opposite of each term in the polynomial you are subtracting and then add. Instead, you can show the polynomial you are subtracting from in a different way so that you can subtract tiles easily.

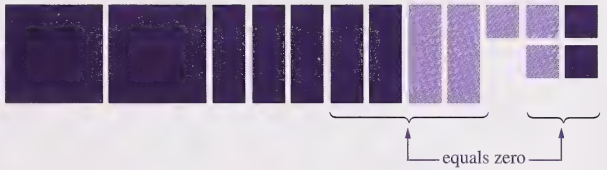
Example 1

Perform the following subtraction of polynomials.

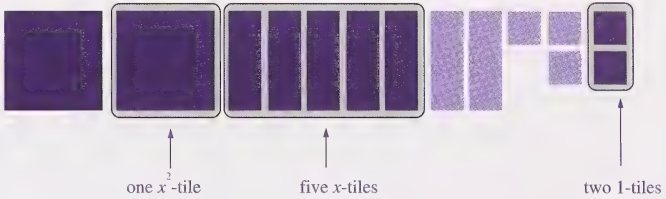
$$\left(2x^2+3x-1\right)-\left(x^2+5x+2\right)$$

Solution

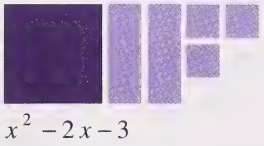
Step 1: You cannot subtract five x-tiles from three x-tiles, nor can you subtract two 1-tiles from one 1-tile. So, you need to represent $2x^2+3x-1$ in a way that allows you to subtract x^2+5x+2. Therefore, add two x-tiles and two $-x$-tiles (which equal zero) and two 1-tiles and two -1-tiles (which equal zero).



Step 2: Now you can remove one x^2-tile, five x-tiles, and two 1-tiles.



Step 3: Show and write an expression for the remaining tiles.



x^2-2x-3

Thus, $\left(2x^2+3x-1\right)-\left(x^2+5x+2\right)=x^2-2x-3$.

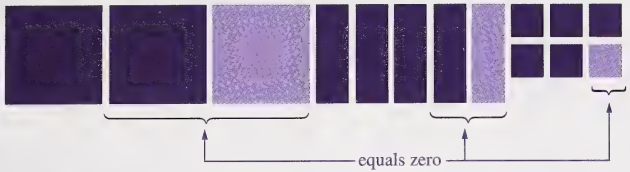
Example 2

Perform the following subtraction of polynomials.

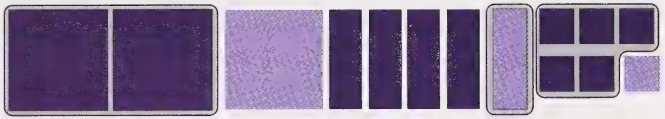
$$\left(x^2+3x+4\right)-\left(2x^2-x+5\right)$$

Solution

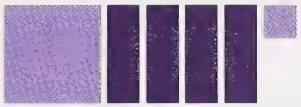
Step 1: Represent x^2+3x+4 in a way that allows you to subtract $2x^2-x+5$. Therefore, add one x^2-tile and one $-x^2$-tile, one x-tile and one $-x$-tile, and one 1-tile and one -1-tile.



Step 2: Now you can remove two x^2-tiles, one $-x$-tile, and five 1-tiles.



Step 3: Show and write an expression for the remaining tiles.



$-x^2+4x-1$

Thus, $\left(x^2+3x+4\right)-\left(2x^2-x+5\right)=-x^2+4x-1$.

1. Use algebra tiles to represent the first polynomial so that you can subtract the second polynomial. Write the expression for the remaining tiles.

a. $(3x+6)-(2x-2)$

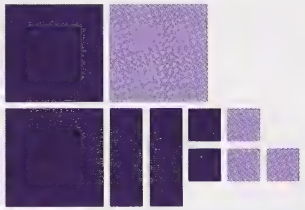
b. $\left(4x^2-3\right)-\left(2x^2+2x+1\right)$

c. $\left(x^2+x+3\right)-\left(2x^2+2x-1\right)$

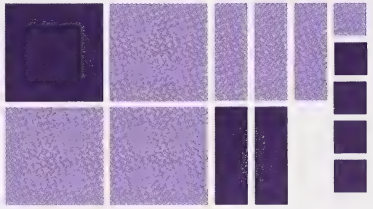
d. $\left(-2x^2+3x+2\right)-\left(x^2-x+1\right)$

2. Give the polynomial represented by each model.

a.



b.



Check your answers by turning to the Appendix.

When you subtract two polynomials without tiles, multiply the sign outside the brackets with each term inside the brackets. Then collect the like terms and perform the required operation.

Example 3

Solve $\left(5x^2+x-2\right)-\left(3x^2-2x+4\right)$.

Solution

$$\begin{aligned}\left(5x^2+x-2\right)-\left(3x^2-2x+4\right) &= 5x^2+x-2-3x^2+2x-4\\ &= 5x^2-3x^2+x+2x-2-4\\ &= (5-3)x^2+(1+2)x+(-2-4)\\ &= 2x^2+3x-6\end{aligned}$$

3. Perform the following subtractions.

a. $\left(3x^2+4x-1\right)-\left(x^2-x+2\right)$

b. $(4x+6)-(2x-1)$

c. $\left(-2x^2+3x+4\right)-(5x-1)$

d. $\left(3x^2+5x\right)-\left(x^2+1\right)$

4. Find the puzzle "What Did the Girl Mushroom Say About the Boy Mushroom After Their First Date?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle. **Remember:** When multiplying a monomial by a polynomial, multiply the monomial by each of the terms in the polynomial.

Example

$$\begin{aligned}&3n^2\left(2n^3+3n^2m-nm^2+4m^3\right)\\ &\quad=3n^2\left(2n^3\right)+3n^2\left(3n^2m\right)+3n^2\left(-nm^2\right)\\ &\qquad+3n^2\left(4m^3\right)\\ &\quad=6n^5+9n^4m-3n^3m^2+12n^2m^3\end{aligned}$$

Check your answers by turning to the Appendix.

If you have access to a computer, you may wish to view the software program *Understanding Algebra* (Neufeld and Associates) for additional help with algebra tiles and adding, subtracting, and multiplying polynomials. Do parts 1 to 6, and answer as many of the Practice questions as you feel are necessary. This program may be available at your school or local library.

Enrichment

In this section, you used algebra tiles to model the product of a polynomial and a monomial. In all cases where models were used, the resulting polynomial was of degree two at most.

1. Suppose you have a question like $x\left(2x^2+x\right)$. Describe how you could model a polynomial of degree three using algebra tiles, and explain why a question like this was not modelled in this section.

Check your answer by turning to the Appendix.

Another way of looking at multiplication of binomials is called the **FOIL** method. This acronym stands for **F**irst terms, **O**utside terms, **I**nside terms, and **L**ast terms.

Multiply the first terms of both binomials to start; then multiply the two outside terms of the binomials; next, multiply the two inside terms of the binomials; and, finally, multiply the last terms of both binomials. Now, add all the like terms and link the terms using the appropriate sign $(+ \text{ or } -)$.

$$(2x+3)(x-4) = \overset{F}{(2x)}(x) + \overset{O}{(2x)}(-4) + \overset{I}{(3)}(x) + \overset{L}{(3)}(-4)$$

$$= 2x^2 - 8x + 3x - 12$$

$$= 2x^2 - 5x - 12$$

Example 1

Multiply $(y-3)$ by $(3y+5)$.

Solution

$$(y-3)(3y+5) = \overset{F}{(y)}(3y) + \overset{O}{(y)}(5) + \overset{I}{(-3)}(3y) + \overset{L}{(-3)}(5)$$

$$= 3y^2 + 5y - 9y - 15$$

$$= 3y^2 - 4y - 15$$

The FOIL method is really just a different way of reminding you to multiply each term of the first binomial by each term of the second binomial.

Do at least six parts of question 2; then do question 3.

2. Find the product for each of the following. Use the FOIL method.

a. $(x+4)(x-3)$

b. $(x-7)(x+2)$

c. $(2x+5)(x+2)$

d. $(3x-1)(2x+5)$

e. $(x-8)(x+8)$

f. $(2x+1)(2x+1)$

g. $(3x+4)(2x-3)$

h. $(x+8)(x-3)$

i. $(2x-5)(x+4)$

j. $(x-10)(x+6)$

k. $(x-9)(x+9)$

l. $(x+7)^2$

3. Simplify the following.

a. $(2y+5)^2$

b. $(y+3)^2-(y+2)(y-4)$

c. $3(y+5)(y-2)$

d. $(x+4)^2-(x-4)^2$

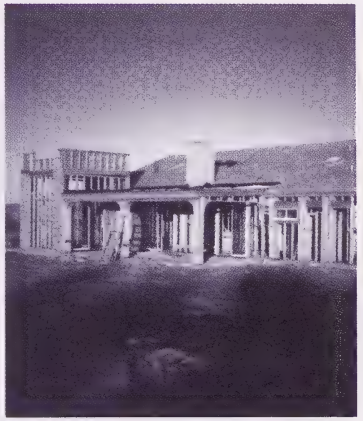
e. $(x+3)(x+5)+2(x+3)$

f. $(x-3)^2+(x+2)(x-2)$

Check your answers by turning to the Appendix.

You may view the video entitled *Multiplying Polynomials* for additional explanation of the FOIL method. This video is available from the Learning Resources Distributing Centre.

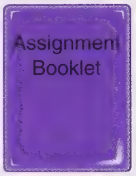
Conclusion



In this section you identified some terminology associated with polynomials and developed some principles on how to model polynomials. You then modelled the four basic operations—addition, subtraction, multiplication, and division—with polynomials using algebra tiles. You also performed these four operations with polynomials algebraically.

Just as a construction worker builds a solid foundation and framework before doing the finishing work on a house, you have now completed the foundation and framework of your knowledge of polynomials. Are you ready to start some finishing work with polynomials in Section 2?

Assignment



You are now ready to complete the assignment for Section 1.

Section 2: Factoring

Have you ever had to help someone find a particular address in a city with named streets? You may have found yourself going down the wrong streets and may have even ended up back where you started. But if you had patience and determination, looked for clues, and learned from your mistakes, you likely were triumphant. Then how do you find your way back to the starting point once you've successfully negotiated your way through? Well, if you've remembered very carefully the route you took to get there, you can retrace your steps by doing everything in reverse.

This is not unlike the situation you find yourself in at this point in the learning about polynomials. You've successfully found your way through the maze of polynomials—learning the correct paths and procedures to follow in order to add, subtract, multiply, and divide polynomials. Now you are being asked to factor polynomials. If you remember the procedures you followed in multiplying polynomials, you should be able to factor polynomials successfully. You are simply going to reverse the procedure, like finding your way back through the subdivision.

Activity 1: Factors and Factoring

In previous mathematics courses you found the factors of numbers (such as 12, 18, and 20). The factors of 12 are 4 and 3, 6 and 2, and 12 and 1; the factors of 18 are 6 and 3, 9 and 2, and 18 and 1; the factors of 20 are 5 and 4, 10 and 2, and 20 and 1. Similarly, polynomials have factors. In this activity you will discover what factors of polynomials are and how to factor simple polynomials using algebra tiles. Using tiles to help understand factoring is similar to using base ten blocks to help understand operations involving decimal numbers.

There are different ways to factor polynomials. You will discover the most efficient and complete methods for factoring in the activities to come.

Before you look at factoring polynomials, you will need to review what is meant by factoring. As you do the following questions, recall some of the things you studied concerning factors and factoring in previous courses. This will give you a good basis upon which to study factoring polynomials.

1. Jeffrey and Jamel have a hobby farm where they raise different breeds of ducks, geese, and chickens. They have enough fencing for a pen with an area of $24\,m^2$.

a. How many different rectangles can you find that each have an area of $24\,m^2$? The dimensions must be whole numbers. List them all.

b. What are these dimensions in relationship to the number 24 (the area of the rectangle)?

2. Copy and complete the following chart in your notebook by filling in the missing dimension of the rectangle.

Length	Width	Area
3 m		$6\,m^2$
6 m		
10 m		$20\,m^2$
	4 m	
	1 m	
16 m		$16\,m^2$
	4 m	
8 m		
5 m		$5\,m^2$

3. What are the factors of each of the following numbers?

a. 6 **b.** 20 **c.** 16 **d.** 5

Check your answers by turning to the Appendix.

To **factor** a number or a polynomial means to write it as a product of two or more factors. If you were asked to factor 12, you could express it as 1×12, 2×6, or 3×4. Therefore, 1, 12, 2, 6, 3, and 4 are factors. If you were asked to factor $8x+6$, you could write it as $2(4x+3)$. Therefore, 2 and $4x+3$ are factors.

Arithmetic	Algebra
Multiply factors to form a product. $4\times3=12$ ← product; factors: 4, 3	**Expand** an expression to form a product. $2(4x+3)=8x+6$; factors: 2, $4x+3$; product: $8x+6$
Factor a number by expressing it as a product of factors. number → $12=4\times3$; factors: 4, 3	**Factor** a polynomial by expressing it as a product of factors. $8x+6=2(4x+3)$; polynomial: $8x+6$; factors: 2, $4x+3$

The operations of multiplying and factoring or expanding and factoring are inverses (or opposites)—they reverse one another.

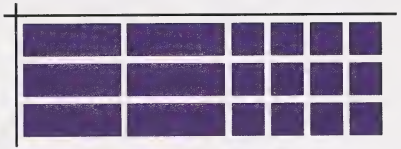
What you are doing in factoring is working backwards (remember the trip along streets with names). Previously, the dimensions were given and you were asked to find the area. Now you will find the dimensions of a rectangle whose area represents the given algebraic expression.

Example 1

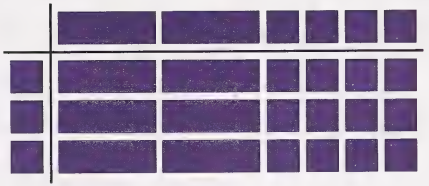
Factor $6x+12$.

Solution

Use the algebra tiles that represent $6x+12$ and arrange them in a rectangle.



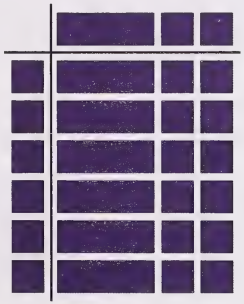
Determine the dimensions of the rectangle.



The dimensions of the rectangle are 3 and $2x+4$. Therefore, the area can be expressed as $3(2x+4)$.

The algebra tiles can also be arranged as follows:

Determine the dimensions of the rectangle.



The dimensions of the rectangle are 6 and $x+2$. Therefore, the area can be expressed as $6(x+2)$.

The tiles could also be arranged end to end—the dimensions would be 1 and $6x+12$. Therefore, the area would be expressed as $1(6x+12)$.

These dimensions (1 and $6x+12$, 3 and $2x+4$, and 6 and $x+2$) are all factors of $6x+12$.

Factoring with 1 and $6x+12$ or 3 and $2x+4$ is incomplete because $6x+12$ and $2x+4$ can both be factored again. For instance, $6x+12=6(x+2)$ and $2x+4=2(x+2)$.

Factoring with 6 and $x+2$ is complete. The polynomial is **completely factored** when you write $6x+12=6(x+2)$. If you look at the rectangular models, the one for $6(x+2)$ is closer to being a square than the others. So, the model that is closest to being a square is the one that will represent the polynomial when it is completely factored. Keep this in mind when you are building the rectangular models of polynomials.

4. Use algebra tiles to show the products (areas) of these polynomials. Write the products algebraically.

a. $3(x+2)$ **b.** $x(x+4)$ **c.** $2(x-1)$ **d.** $2x(x+2)$

5. Using the models from question 4, what are the factors of each of the following polynomials?

a. $3x+6$ **b.** x^2+4x **c.** $2x-2$ **d.** $2x^2+4x$

6. If you know one factor of a polynomial, how can you find the other one without using algebra tiles?

Example 2

Determine the factors of $5x^2 - 10x$ if one factor is $5x$.

Solution

$$(5x^2 - 10x) \div 5x = \frac{5x^2}{5x} - \frac{10x}{5x}$$
$$= x - 2$$

The factors are $5x$ and $(x-2)$.

7. Find the missing factor in each.

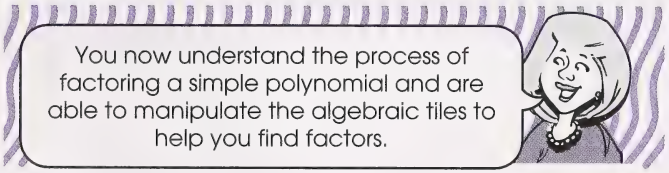
a. $4x + 6 = 2(\quad)$ **b.** $x^2 + 3x = x(\quad)$

c. $5x - 10 = 5(\quad)$ **d.** $2x^2 + 6x = 2x(\quad)$

e. $2x^2 + 4x + 10 = 2(\quad)$

f. $4x^2 + 8x + 12 = 4(\quad)$

Check your answers by turning to the Appendix.



Activity 2: Common Factors

What do all these people have in common? If you said they have all been president of the United States, then you are right.

When you are asked to factor a polynomial, the first thing you should look for is any factor that every term in the polynomial has in common. Recall, from previous mathematics courses, that a common factor of 12, 24, and 30 is 6. Likewise, a common factor of $6a^2$, $9a$, and $3a^3$ is $3a$. When you factor a polynomial like $4x + 6$, you get $2(2x + 3)$. The 2 is called a **common factor**. That is because it is a factor of both terms in the polynomial.

Example 1

Factor $3x + 6$ using algebra tiles.

Solution

Form a rectangle with the tiles.

Draw in the dimensions.

The length is $x+2$ and the width is 3. Therefore, the area as a product is $3(x+2)$. **Note:** The common factor in this example is 3.

Example 2

Factor $2x-4$ using algebra tiles.

Solution

Form a rectangle with the tiles.

Draw in the dimensions.

The length is $x-2$ and the width is 2. The area, as a product, is $2(x-2)$. **Note:** The common factor in this example is 2.

1. Copy and complete the following chart in your notebook. Use algebra tiles to form rectangles, and read the length from across the top of the rectangle and the width down the left side.

Area	Length	Width	Area as a Product
$2x+8$			
$3x-6$			
$6x+2$			
$4x+8$			
$6y+3$			
$8y-4$			

2. Which dimension represents the common factor in each polynomial in question 1?

3. Use algebra tiles to help factor each of the following.

a. $4x+2$ **b.** $6x+9$

c. $6x-3$ **d.** $2x^2+4x$

e. x^2+3x **f.** $2x^2-2x$

4. What is the common factor of each polynomial in question 3?

Check your answers by turning to the Appendix.

When you are asked to factor a polynomial, you must factor it completely. As you have seen from some of the examples, there are often several ways to factor a polynomial; but to factor completely, you must find the **greatest common factor (GCF)**.

The greatest common factor of a set of terms is the largest term that will divide evenly into every term of the set—that is, the GCF of the numerical coefficients and the GCF of the variables of the terms in the polynomial.

5. After you have found the greatest common factor, how do you find the other factor?

Check your answer by turning to the Appendix.

You may recall from previous mathematics courses that the GCF of a set of three numbers is the largest number that will divide evenly into each number of the set.

Example 3

Find the GCF of 24, 18, and 30.

Solution

First, factor each number.

$$24 = 2 \times 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$30 = 2 \times 3 \times 5$$

Now, find the common factors, and multiply to find the GCF.

$$24 = 2 \times 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$30 = 2 \times 3 \times 5$$

$$\text{GCF} = 2(3)$$
$$= 6$$

Note: The largest number that will divide into 24, 18, and 30 is 6; so, 6 is the GCF.

Now look at an example that involves a variable.

Example 4

What is the GCF of $2x^3$, $8x^2$, and $-4x$?

Solution

Factor each term, and find the common factors.

$$2x^3 = 2 \times x \times x \times x$$
$$8x^2 = 2 \times 2 \times 2 \times x \times x$$
$$-4x = -2 \times 2 \times x$$

Therefore, the GCF is $2(x) = 2x$.

You can extend this method to factor a trinomial.

Example 5

What are the factors of $2x^3 + 8x^2 - 4x$?

Solution

Using the information from Example 4, the GCF of the three terms is $2x$. To find the other factor, divide each term by $2x$.

$$\frac{2x^3 + 8x^2 - 4x}{2x} = \frac{2x^3}{2x} + \frac{8x^2}{2x} - \frac{4x}{2x}$$
$$= x^2 + 4x - 2$$

Therefore, the factors of $2x^3 + 8x^2 - 4x$ are $2x$ and $x^2 + 4x - 2$.

Verification

$$2x\left(x^2 + 4x - 2\right) = 2x\left(x^2\right) + 2x(4x) + 2x(-2)$$
$$= 2x^3 + 8x^2 - 4x$$

Example 6

Factor $3a^2b - 6ab^2 + 9ab$ completely.

Solution

Find the GCF of the three terms.

$$3a^2b = 3 \times a \times a \times b$$
$$-6ab^2 = -2 \times 3 \times a \times b \times b$$
$$9ab = 3 \times 3 \times a \times b$$

The GCF is $3(a)(b) = 3ab$.

To find the other factor, divide the polynomial by $3ab$.

$$\frac{3a^2b - 6ab^2 + 9ab}{3ab} = \frac{3a^2b}{3ab} + \frac{-6ab^2}{3ab} + \frac{9ab}{3ab}$$
$$= a - 2b + 3$$

Therefore, $3a^2b - 6ab^2 + 9ab$ factored completely is $3ab(a - 2b + 3)$.

As you have seen from some of the examples, it is not always possible to use the tiles to represent polynomials and, therefore, use them to find the factors. You must know how to find the GCF and then divide to find the other factor.

6. Find the greatest common factor of the following sets of numbers or terms.

a. $15, 20, 35$

b. $12, -16, 20$

c. $6x, 8x^2, 4x^3$

d. $-9x, 15x^2, -12x^3$

e. $24z^4, 30z^3, -12z^2$

f. $9ab^2c, 12a^2b, 21b^2c$

7. Find the missing factor.

a. $4a^2+6a=2a(\quad)$

b. $7a^3-14a^2+28a=7a(\quad)$

c. $18b+3b^2=\quad(6+b)$

d. $12b^2+20b^3-24b^4=\quad\left(3+5b-6b^2\right)$

e. $10b^3+8b^2-12b+6=2(\quad)$

8. Factor each of the following. Verify your answers by expanding.

a. $2x^2+8x$

b. $4ab^2-7a^2b$

c. $4a^2+6a+8$

d. $7y^2+14y^3$

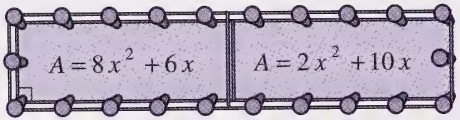
e. $4a-16$

f. $2xy+6x^2y^2-8x^3y^3$

g. $ab+ac$

h. $ab+ac-2ad+4ae$

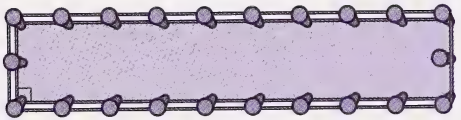
9. Jeffrey and Jamel have a hobby farm on which they raise ostriches. Jeffrey and Jamel have two attached rectangular pens with the given areas as shown.



Show how you find the length of the common side.

10. The formula for the perimeter of a rectangle is $P=2\ell+2w$. Factor this formula.

11. The area of another rectangular pen on Jeffrey and Jamel's hobby farm is $3x^2+x$. Find its dimensions.



Check your answers by turning to the Appendix.

Common Binomial Factors

Sometimes the greatest common factor is a binomial. The method used to factor these expressions is very similar to the method used to factor expressions with common monomial factors.

Example 1

Factor $x(x+3)+2(x+3)$.

Solution

$x+3$ is a common factor in each term. It is multiplied by x and by 2.

$$\therefore\ x(x+3)+2(x+3)=(x+3)(x+2)$$

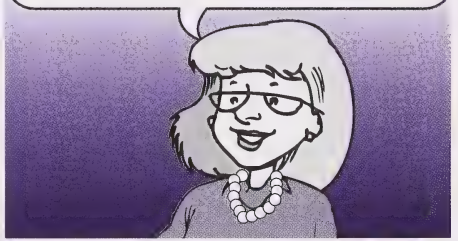
Verification

Expand both sides.

LS	RS
$x(x+3)+2(x+3)$	$(x+3)(x+2)$
$=(x)(x)+(x)(3)+(2)(x)+(2)(3)$	$=(x)(x)+(x)(2)+(3)(x)+(3)(2)$
$=x^2+3x+2x+6$	$=x^2+2x+3x+6$
$=x^2+5x+6$	$=x^2+5x+6$

LS = RS

The examples used here have the terms grouped so the common binomial factor is obvious. This is not always the case. For more explanation, study the examples and explanations in the Enrichment part of this section.



Example 2

Factor $a(a-2)+b(a-2)$.

Solution

The common binomial factor is $a-2$.

$$\therefore\ a(a-2)+b(a-2)=(a-2)(a+b)$$

Verification

Let $a=4$ and $b=5$. Substitute these values into each expression and evaluate.

LS	RS
$a(a-2)+b(a-2)$	$(a-2)(a+b)$
$=4(4-2)+5(4-2)$	$=(4-2)(4+5)$
$=4(2)+5(2)$	$=(2)(9)$
$=8+10$	$=18$
$=18$	

LS = RS

12. Identify the common binomial factor in each of the following expressions.

a. $(m+n)a+(m+n)b$ **b.** $3c(a-3)+2(a-3)$

c. $y(2x+3)+1(2x+3)$ **d.** $(d-1)x+(d-1)y$

e. $a(a+3)-b(a+3)$ **f.** $y(x-1)-3(x-1)$

13. Find the common binomial factor of each expression, and factor completely.

a. $x(x-7)+3(x-7)$ **b.** $2y(y+3)-1(y+3)$

c. $a(a+7)+3(a+7)$ **d.** $(2n+3)n+(2n+3)3$

e. $(m+1)m-(m+1)3$ **f.** $2t(t+3)+5(t+3)$

14. Factor completely. Do at least five.

a. $(a+3)b-(a+3)c$ **b.** $2y(y+4)+3(y+4)$

c. $2a(b+1)+c(b+1)+2(b+1)$ **d.** $(z-y)6+(z-y)x$

e. $(xy+2y)+(2x+4)$ **f.** $\left(a^2+8a\right)-(3a+24)$

g. $x^2+5x+6x+30$ **h.** $y^2+4y+3y+12$

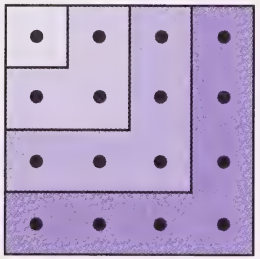
Check your answers by turning to the Appendix.

Now Try This

Use one of the problem-solving strategies to solve the following problems.

15. a. Find the sum of $1+3+5+7$.

b. Using the pattern suggested by the diagram, find the sum of $1+3+5+7+\ldots+15$.



c. What is the sum of the first twenty odd numbers?

d. If n represents the number of odd numbers being added, write an expression for the last odd number.

e. Find the rule for the sum of $1+3+5+7+\ldots+(2n-1)$.

Check your answers by turning to the Appendix.

Factorially Yours

Factorial means the product of all the positive integers less than or equal to a particular number. Factorial n is denoted by the symbol $n!$. For example, $4!$ means $4\times3\times2\times1$.

Use a calculator to answer question 16.e.

16. a. What is the value of $5!$?

b. What is $7\times6!$ the same as?

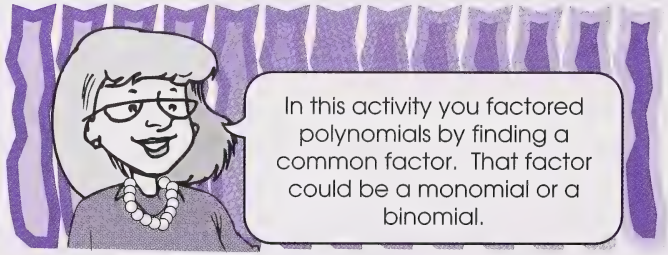
c. What is the value of $5!\div4!$?

d. Find the value of $\frac{10!\times8!}{9!\times7!}$.

e. Find the value of each of the following.

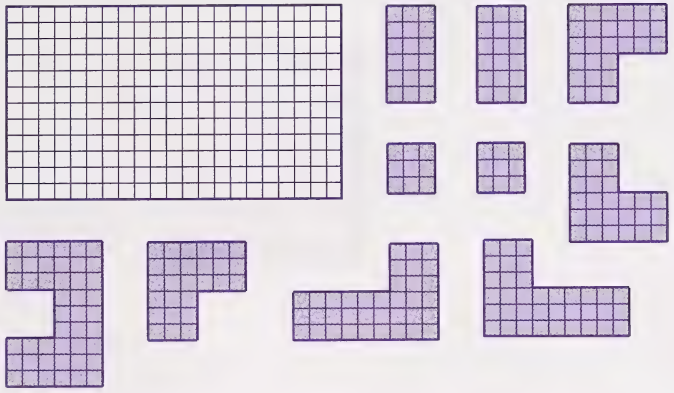
i. $(7-1)!$ ii. $\left(\sqrt{100}\right)!$ iii. $8!-6!$

Check your answers by turning to the Appendix.



Activity 3: Factoring Trinomials of the Form $x^2 + bx + c$

Can you fill the rectangle with the shapes provided? Find the grid at the end of the Appendix; cut out the rectangle and the pieces; and arrange the pieces to fill in the rectangle. You should be able to fill the space in the rectangle with no pieces left over. The grid and the pieces look as follows:



Solving this puzzle is similar to the process used in factoring trinomials using algebra tiles. You use the algebra tiles to represent the trinomial and then arrange the tiles to form a rectangle. The factors of the trinomial are the lengths of the sides of the rectangle.

Watch the video entitled *Freddie Factor*. Use algebra tiles to do the video assignment. **Note:** You are not responsible for trinomials of the type $ax^2 + bx + c$, so it is not necessary to do that part of the video assignment.

Check your answers to the video assignment by viewing the video.

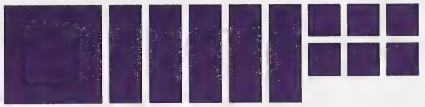
Were you able to do the questions involving trinomials of the form $x^2 + bx + c$ in the video assignments? If you had any difficulties doing the video questions, work through the next three print examples. Use algebra tiles to help you follow the examples. If you had no difficulties, you may wish to go directly to the exercises following Example 3.

Example 1

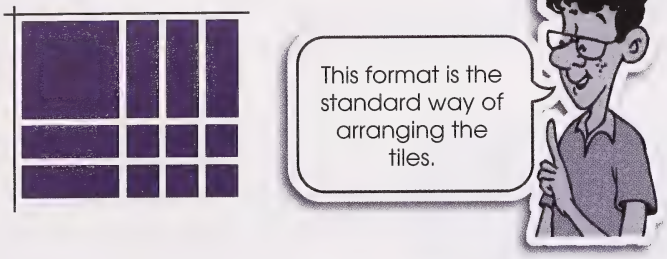
Factor the trinomial $x^2 + 5x + 6$.

Solution

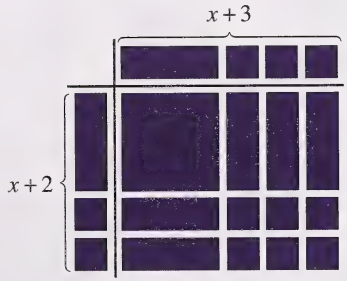
First, model the trinomial.



Next, arrange the tiles to form a rectangle. Place the x^2-tiles on the upper left and the 1-tiles on the lower right.



The factors of the trinomial are represented by the length and width of the rectangle.



1. What is the length and width of the rectangle?

2. Write the factors of the trinomial x^2+5x+6.

Check your answers by turning to the Appendix.

To verify the factors of the trinomial in Example 1, multiply them.

$$(x+3)(x+2) = x^2+2x+3x+6$$
$$= x^2+5x+6$$

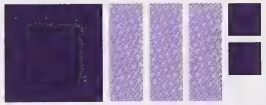
The resulting product is the same as the original trinomial; thus, the factors are correct and $x^2+5x+6=(x+3)(x+2)$.

Example 2

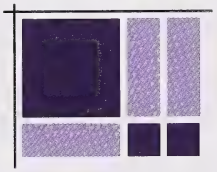
Factor the trinomial x^2-3x+2.

Solution

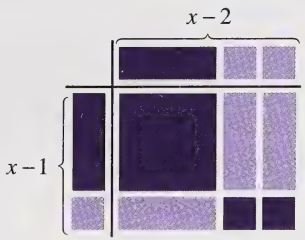
First, model the trinomial.



Next, arrange the tiles to form a rectangle.



Place the tiles representing the length and the width.



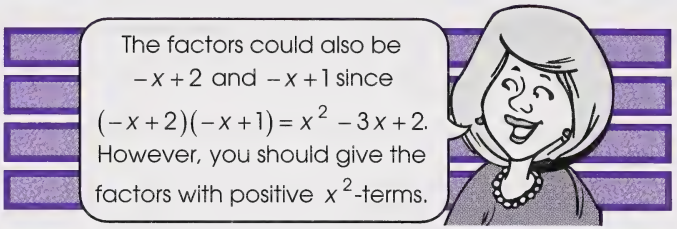
3. Write the length and width of the rectangle, and state the factors of the trinomial $x^2 - 3x + 2$.

Check your answer by turning to the Appendix.

Multiply to verify the solution to Example 2.

$$(x-2)(x-1) = x^2 - 1x - 2x + 2$$
$$= x^2 - 3x + 2$$

Therefore, $x^2 - 3x + 2 = (x-2)(x-1)$.

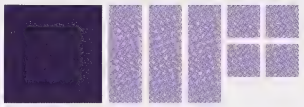


Example 3

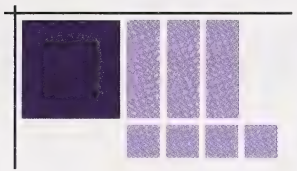
Factor the trinomial $x^2 - 3x - 4$.

Solution

First, model the trinomial.

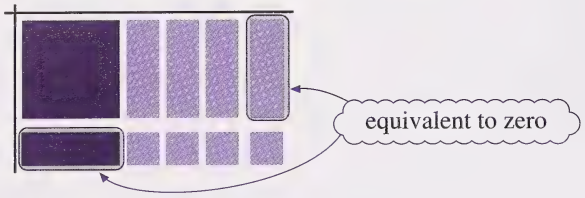


Next, arrange the tiles to form a rectangle.

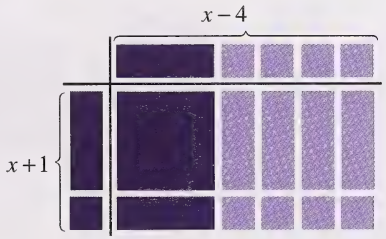


Notice that it is impossible to arrange them to form a rectangle without using more tiles. You can add more tiles without changing the trinomial by applying the zero principle.

Add one x-tile and one $-x$-tile to complete the rectangle.



Draw the tiles representing the length and width.



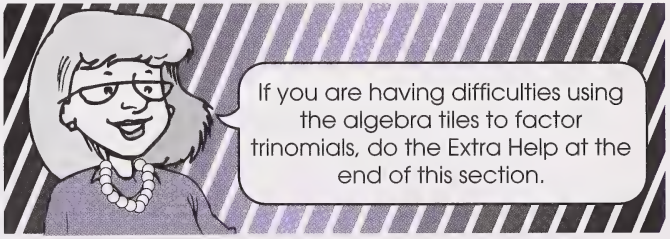
4. Write the length and width of the rectangle, and state the factors of the trinomial $x^2 - 3x - 4$.

Check your answer by turning to the Appendix.

Multiply to verify the solution to Example 3.

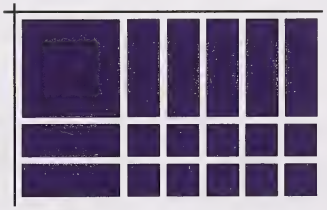
$$(x-4)(x+1) = x^2 + x - 4x - 4$$
$$= x^2 - 3x - 4$$

Therefore, $x^2 - 3x - 4 = (x-4)(x+1)$.



Now you are ready to try some exercises involving factoring of trinomials using algebra tiles.

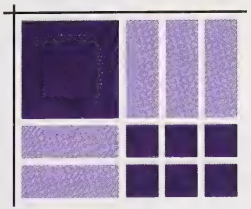
5. a. Write the trinomial represented by the rectangular model.



b. Use the rectangular model to factor the trinomial.

c. Verify your answer to question 5.b.

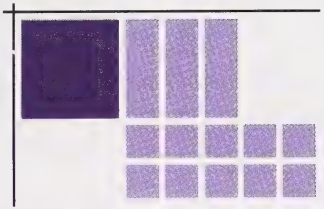
6. a. Write the trinomial represented by the rectangular model.



b. Use the rectangular model to factor the trinomial.

c. Verify your answer to question 6.b.

7. a. Write the trinomial represented by the model.



b. Add tiles to complete the rectangle; then use the rectangular model to factor the trinomial.

c. Verify your answer to question 7.b.

8. Use the algebra tiles to factor the following trinomials.

a. $x^2 + 8x + 12$ **b.** $x^2 + 8x + 15$

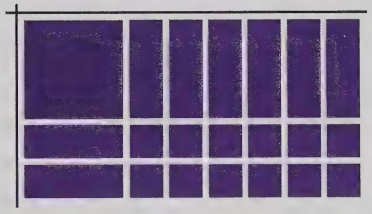
c. $x^2 - 5x + 4$ **d.** $x^2 - 6x + 8$

e. $x^2 - 4x - 5$ **f.** $x^2 + 2x - 8$

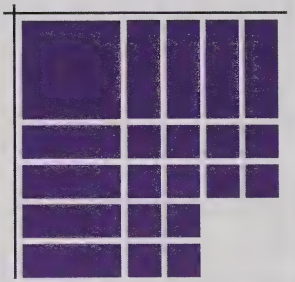
Study the following information to answer question 9.

Annette and Krish are using algebra tiles to factor the polynomial $x^2 + 8x + 12$. Their arrangements are as follows:

Annette's



Krish's



9. Explain why Annette's arrangement is correct and Krish's is not.

Check your answer by turning to the Appendix.

Inspect the binomial factors of a trinomial and you should be able to see some patterns that will allow you to factor a trinomial without having to use algebra tiles.

Look at the following three trinomials, and their factors and note the patterns.

$$n^2 + 7n + 12 = (n+4)(n+3)$$

$+7 = 4+3$ sum

$+12 = 4 \times 3$ product

$$n^2 - 6n + 5 = (n-5)(n-1)$$

$-6 = (-1)+(-5)$ sum

$+5 = (-1)\times(-5)$ product

$$n^2 + 3n - 10 = (n+5)(n-2)$$

$+3 = 5+(-2)$ sum

$-10 = 5\times(-2)$ product

10. State the pattern that you find in the previous three trinomials.

Check your answer by turning to the Appendix.

To factor a polynomial of the form $x^2 + bx + c$, you need to find two numbers whose product is c and whose sum is b.

For instance, to factor a trinomial like $x^2 + 10x + 16$, you need to find two numbers whose product is 16 and whose sum is 10.

11. Write out all the factors of 16.

12. Which pair of factors add up to 10?

Check your answers by turning to the Appendix.

The numbers 8 and 2 become the second terms in the factors of the trinomial. Thus, the factors are $x+8$ and $x+2$.

You can verify the factors by multiplying.

$$\begin{aligned}(x+8)(x+2) &= x^2 + 2x + 8x + 16 \\ &= x^2 + 10x + 16\end{aligned}$$

Therefore, $x^2 + 10x + 16 = (x+8)(x+2)$.

Example 4

Factor the trinomial $y^2 - y - 20$.

Solution

You need to find two numbers whose product is -20 and whose sum is -1. The factors of -20 are as follows:

- -20 and 1 ← The sum is – 19.
- 20 and -1 ← The sum is 19.
- -10 and 2 ← The sum is – 8.
- 10 and -2 ← The sum is 8.
- -5 and 4 ← The sum is – 1.
- 5 and -4 ← The sum is 1.

The correct numbers are -5 and 4, thus becoming the second terms in the binomial factors. The factors are $y-5$ and $y+4$.

Verification

$$(y-5)(y+4) = y^2 + 4y - 5y - 20$$
$$= y^2 - y - 20$$

Therefore, $y^2 - y - 20 = (y-5)(y+4)$.

13. Copy and complete the factoring of the following trinomials by filling in the proper sign ($-$ or $+$) in the binomial factors. Check your answers.

a. $x^2 + 11x + 30 = (x \quad 5)(x \quad 6)$

b. $x^2 - 9x + 20 = (x \quad 5)(x \quad 4)$

c. $n^2 - 3n - 18 = (n \quad 6)(n \quad 3)$

d. $y^2 + 5y - 14 = (y \quad 7)(y \quad 2)$

14. Find the missing binomial factor for each of the following.

a. $x^2 + 13x + 36 = (x+4)(\quad)$

b. $x^2 - 14x - 32 = (x-16)(\quad)$

c. $y^2 - 11y + 24 = (y-3)(\quad)$

15. Factor the following trinomials. Verify your answers.

a. $x^2 - 12x + 20$

b. $n^2 - 4n - 12$

c. $y^2 + 11x - 12$

d. $m^2 - 6m + 9$

e. $p^2 + 8p + 16$

f. $b^2 + b - 56$

Check your answers by turning to the Appendix.

You have now factored polynomials by finding the greatest common factor (GCF), and you have factored trinomials into the product of two binomials. Sometimes you will meet trinomials that require you to apply both of these factoring techniques in order to factor them completely.

Example 5

Factor $2x^2 + 22x + 56$.

Solution

First, factor by taking out the common factor of 2.

$$2x^2 + 22x + 56 = 2\left(x^2 + 11x + 28\right)$$

Next, factor the trinomial by finding two numbers whose product is 28 and whose sum is 11. Since the sum is positive, only the positive factors of 28 need to be considered.

The positive factors of 28 are as follows:

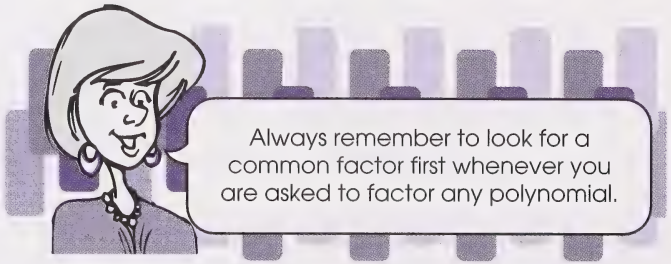
- 28 and 1 ← The sum is 29.
- 14 and 2 ← The sum is 16.
- 7 and 4 ← The sum is 11.

The correct numbers are 7 and 4. Thus, the factors of $x^2 + 11x + 28$ are $(x+7)$ and $(x+4)$.

Verification

$$\begin{aligned} 2(x+7)(x+4) &= 2\left(x^2 + 4x + 7x + 28\right) \\ &= 2\left(x^2 + 11x + 28\right) \\ &= 2x^2 + 22x + 56 \end{aligned}$$

Therefore, $2x^2 + 22x + 56 = 2(x+7)(x+4)$.



16. Factor the following trinomials. Find the GCF first.

a. $3x^2 + 15x + 12$

b. $2x^2 - 20x + 50$

c. $2x^3 + 14x^2 + 12x$

d. $3x^3 + 21x^2 - 54x$

e. $6y^2 + 12y + 6$

f. $x^2y + 3xy - 40y$

g. $4x^2 - 28x + 48$

h. $x^3 + 2x^2 + x$

Check your answers by turning to the Appendix.

Now Try This

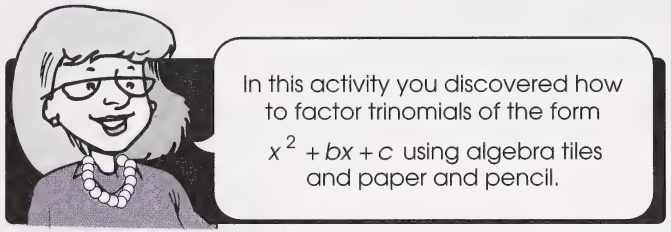
Solve the following challenge problem.

17. **a.** Using algebra tiles, display $x^2 + 4x - 5$ as a rectangle.

b. What are the dimensions of the rectangle?

c. Explain how algebra tiles are used to find the factors of a polynomial, where the tiles do not form a complete rectangle at the start.

Check your answers by turning to the Appendix.



Follow-up Activities

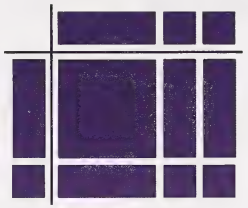
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may do both.

Extra Help

Factoring means to separate a number or polynomial into a product of two or more parts. Factoring trinomials means to rewrite the trinomial as a product of two binomials.

The following examples will give you more practise with using algebra tiles to factor trinomials of the form $x^2 + bx + c$.

Recall, from Section 1, multiplying a binomial by a binomial using algebra tiles. For example, multiply $(x+1)$ by $(x+2)$.



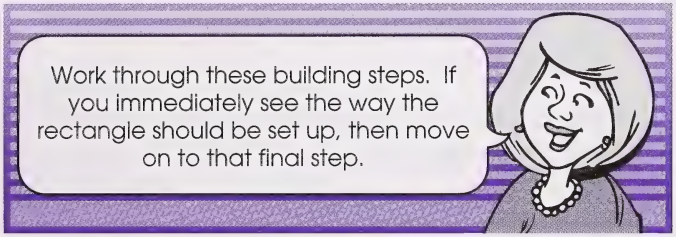
Therefore, $(x+1)(x+2) = x^2 + 3x + 2$.

Since the factoring of polynomials undoes multiplication, you can start with the products and work backward to the factors. Actually, this is quite simple since all you have to do is arrange the tiles related to the trinomial as a **rectangle**. Once you have made the rectangle, count the different tiles, put them together as a sum or difference, and read the factors at the side and at the top.

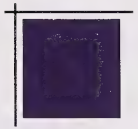
Example 1

Factor $x^2 + 5x + 6$ using algebra tiles.

Solution



Since there is only one x^2-term, you can have the x^2-tile in only one place.

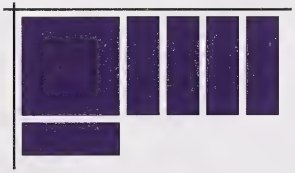


Now you have five x-tiles to place in such a way that a rectangle is formed. There are a few possibilities. One possibility is having all five tiles placed straight across from the x^2-tile.

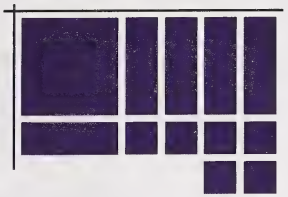
If you choose this arrangement, you now have to place six 1-tiles in such a way that the result is a rectangle. You cannot place them in the following way since there is no rectangle after all placements are done.

Therefore, the previous arrangement cannot be used.

Try putting the x-tiles into this pattern.



When placing the 1-tiles, you get an arrangement like this.

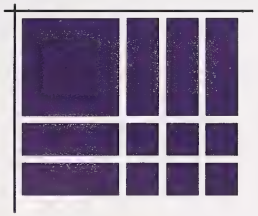


You have two 1-tiles left over and you still do not have a rectangle.

Another starting possiblity is this.

This is a good choice since 3 and 2 are factors of 6 that add up to 5.

Now put the 1-tiles in place to finally get a rectangle.

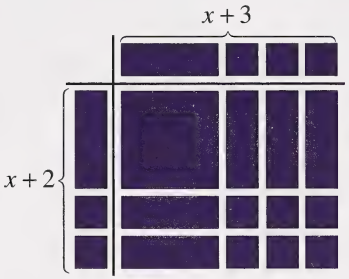


Do you think this is the answer? To verify the diagram, count the tiles to see if there are enough of each shape.

Do you have one x^2-tile, five x-tiles, and six 1-tiles? Yes. Do you have a rectangle? Yes.

All of the required conditions are met; so, you can now proceed to find the actual factors.

Draw in the factors as shown. The shapes along the side and along the top are now counted to determine the factors.



Therefore, the factors are $(x+2)$ and $(x+3)$.

Verification

$$(x+2)(x+3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Since the product is the same as the original polynomial, the factors are correct.

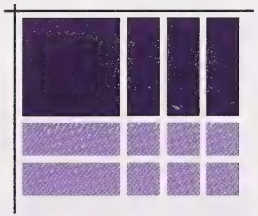
Study another example.

Example 2

Factor $x^2 + x - 6$.

Solution

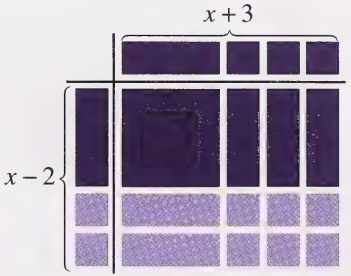
Is it possible to make a rectangle with exactly one x^2-tile, one x-tile, and six 1-tiles? The answer is no. However, the following arrangement of algebra tiles represents $x^2 + x - 6$.



How does this arrangement of algebra tiles represent $x^2 + x - 6$?

You know that you need one x^2-tile for x^2. Since $-2x + 3x = x$, you can use two $-x$-tiles and three x-tiles to make x. You also need six -1-tiles for -6.

Draw in the factors.



Therefore, the factors are $(x-2)$ and $(x+3)$.

Verification

$$(x-2)(x+3) = x^2 + 3x - 2x - 6$$
$$= x^2 + x - 6$$

Do at least four of the following questions. Do the rest if you want more practice.

1. Factor the following using algebra tiles.

a. $x^2 + 7x + 12$

b. $x^2 - x - 2$

c. $x^2 - 3x + 2$

d. $x^2 + x - 6$

e. $x^2 - 3x - 4$

f. $x^2 + 4x - 5$

2. Find the puzzle "Where Do Tadpoles in the Pawn Shop Come From?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

Check your answers by turning to the Appendix.

If you have access to a computer, you may wish to view the software program *Understanding Algebra* (Neufeld and Associates) for additional help with factoring. Do Part 7: Factoring Expressions, and answer as many of the Practice questions as you feel are necessary. This program may be available at your school or local library.

Enrichment

In Activity 2, you looked at some expressions that had a common binomial factor. The common factor should have been very visible to you, with the exception of some questions in the last exercise.

Example 1

Factor $m(m+3)+4(m+3)$.

Solution

The common factor is $m+3$.

$\therefore\ m(m+3)+4(m+3)=(m+3)(m+4)$

Now you will look at how you can factor expressions by grouping the terms.

Example 2

Factor $m^2+3m+4m+12$.

Solution

Group the terms in pairs so that each member of the pair has a common factor.

$$\left(m^2+3m\right)+(4m+12)$$

This pair has a common factor of m.

This pair has a common factor of 4.

Now, factor the common terms (m and 4), from each of the two groups respectively.

$$\left(m^2+3m\right)+(4m+12)$$
$$=m(m+3)+4(m+3)$$

You now have a common binomial factor of $m+3$.

Therefore,

$m^2+3m+4m+12=(m+3)(m+4)$.

Example 3

Factor $ab+3xy+ay+3bx$.

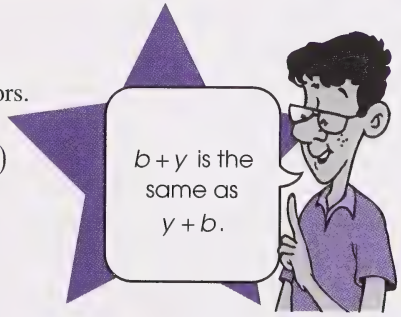
Solution

You have to regroup the terms before you can see the common factors.

$$ab+ay+3xy+3bx$$

Now factor the common factors.

$$(ab+ay)+(3xy+3bx)$$
$$=a(b+y)+3x(y+b)$$
$$=(b+y)(a+3x)$$



Note: You could have grouped $(ab+3bx)$ and $(ay+3xy)$.

$$(ab+3bx)+(ay+3xy)=b(a+3x)+y(a+3x)$$
$$=(a+3x)(b+y)$$

1. Factor completely by grouping.

a. $a^2-6a+2a-12$

b. $ab^2+abc+db+dc$

c. $xy+3x-y-3$

d. $x^2+15+3x+5x$

e. $ma-2n+na-2m$

f. $m^2-2m-5m+10$

g. $abc+adc+4be+4de$

h. $ad+bd+2ac+2bc$

i. $n^2+11n+24$

Check your answers by turning to the Appendix.

You may have had some difficulty with the last expression in question 1. There are only three terms, so how can you group them into two pairs? This is a slightly different kind of polynomial that can still be solved by grouping. You just have an extra step added to the process.

Example 4

Factor $n^2+11n+24$.

Solution

Rewrite the middle term as two terms whose sum is $11n$ and whose coefficients have a product of 24.

$$n^2+3n+8n+24$$

$3n+8n=11n$
$3\times 8=24$

Now you can group and factor.

$$(n^2+3n)+(8n+24)=n(n+3)+8(n+3)$$
$$=(n+3)(n+8)$$

Example 5

Factor $y^2 + 9y + 18$.

Solution

Rewrite the middle term.

$$y^2 + 3y + 6y + 18$$

$3y + 6y = 9y$
$3 \times 6 = 18$

Group and factor.

$$\left(y^2 + 3y\right) + (6y + 18) = y(y + 3) + 6(y + 3)$$
$$= (y + 3)(y + 6)$$

2. Factor the following polynomials completely using the grouping method. Do at least seven.

a. $y^2 + 9y + 14$

b. $x^2 - 7x + 12$

c. $a^2 + 13a + 36$

d. $n^2 + 9n + 20$

e. $b^2 - 10b + 21$

f. $c^2 - 2c - 15$

g. $m^2 + 12m + 27$

h. $a^2 - 13a + 30$

i. $y^2 + 9y + 8$

j. $x^2 + 9x - 22$

Check your answers by turning to the Appendix.

Conclusion

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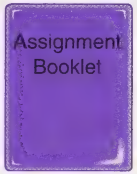
In this section, you began the process of learning how to factor polynomials by reviewing what is meant by factors and factoring. You used algebra tiles to factor simple polynomials, and extended this to finding common factors for monomials and polynomials. You took another step further when you examined expressions that had a common binomial factor. Finally, you factored trinomials into two binomial factors.

You should now have a good grasp of the basics of factoring polynomials, which you will expand upon as you move into future mathematics courses.

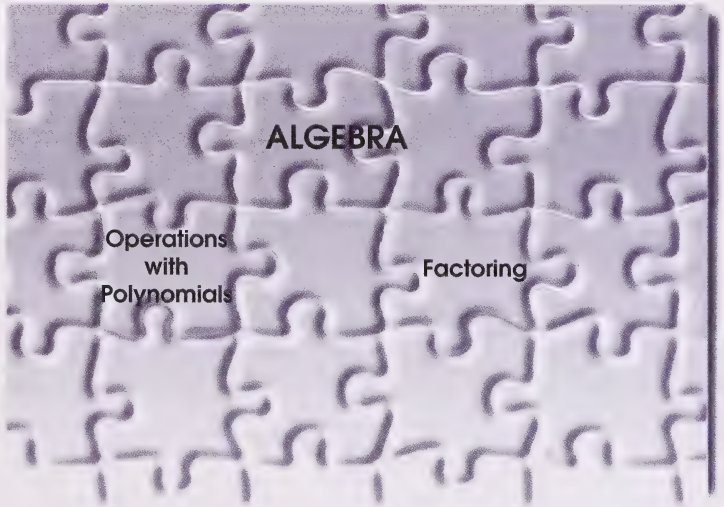
When working with polynomials, you took note of how to multiply polynomials to help you discover factoring (the reverse procedure of multiplying. This is similar to finding your way in and out of a city subdivision. You learn the route by taking note of important details and special clues along the way. After completing this module, do you feel comfortable working your way in and out of the subdivision called polynomials?

Assignment



You are now ready to complete the assignment for Section 2.

Module Summary

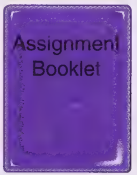


The algebra puzzle is now a couple of pieces closer to completion. By completing this module, you have fit two more pieces (concepts)—Operations with Polynomials and Factoring—into your algebra puzzle.

You identified polynomials, discovered the functions of polynomials, and discovered how to work with polynomials. You also used algebra tiles to model polynomials and determine how to add, subtract, multiply, divide, simplify, and factor polynomials. You then used your modelling experience to determine how to perform these operations using only paper and pencil.

Module 4 has broadened your knowledge of algebra by introducing you to the world of polynomials. It has provided information necessary for you to proceed in finding other pieces to the algebra puzzle in future mathematics courses.

Final Module Assignment



You are now ready to complete the final module assignment.

APPENDIX

- Glossary
- Suggested Answers
- Puzzles
- Cut-out Learning Aids

Glossary

Ascending order: in order from smallest to largest

Binomial: a polynomial with two terms

Common factor: a factor that is common to two or more numbers or terms

Constant: an algebraic term that consists of a number by itself

Degree of a monomial: the sum of the exponents of all of the variables

$4xy^2$ is of degree 3.

Degree of a polynomial: the degree of the term that has the largest sum of exponents

$3xy + 4x^2y^2 + 9x - 4$ is of degree 4.

Descending order: in order from largest to smallest

Distributive property: a rule for expanding an expression within brackets by multiplying each term inside the brackets by the term outside the brackets

$4(7+6) = (4\times 7) + (4\times 6)$ $\qquad$ $3(n-2) = 3n - 6$

Expanding: removing the brackets and simplifying an expression

$3\left(x^2 + 2x - 3\right) = 3x^2 + 6x - 9$

Factor: to write a number or polynomial as a product

$8 = 2\times 2\times 2$ $\qquad$ $4m^2n = (4)(m)(m)(n)$

Factorial: the product of all the positive integers less than or equal to a particular number

$4! = 4\times 3\times 2\times 1 = 24$

FOIL method: an acronym for a method of multiplying binomials—**F**irst terms, **O**utside terms, **I**nside terms, **L**ast terms

Greatest common factor (GCF): the largest factor that a group of numbers or terms has in common

The GCF of $12x^3y$, $16x^2y^2$, and $8xyz$ is $4xy$.

Like terms: terms with identical literal coefficients

$4x^2y$, $-6x^2y$, and x^2y are like terms.

Literal coefficient: a non-numerical factor (or factors) of an algebraic term

Mathematical expression: a combination of numerals, variables, and other mathematical symbols

$4x \qquad 3n^2 + 4n - 1 \qquad \frac{x+3}{x-4}$

Monomial: a polynomial with one term

Numerical coefficient: the numerical factor of an algebraic term

Polynomial: one term or a combination of terms added together

Simplest form: a fraction written so the numerator and denominator have no common factors

$\frac{10}{12} = \frac{5}{6} \qquad \frac{4xy}{6xz} = \frac{2y}{3z}$

Term: a mathematical expression containing the product of only numbers and/or algebraic symbols

$4a^2b,\ 15n,\ -5mn$

Trinomial: a polynomial with three terms

Variable: a letter that represents a number

Zero principle: the principle that two opposites cancel one another or result in zero

Suggested Answers

Section 1: Activity 1

1. a. The numerical coefficient is 7.
The literal coefficient is ab.
The variables are a and b.

b. The numerical coefficient is -8.
The literal coefficient is y^3.
The variable is y.

c. The numerical coefficient is 4.
The literal coefficient is xyz.
The variables are x, y, and z.

d. The numerical coefficient is -6.
The literal coefficient is z.
The variable is z.

e. The numerical coefficient is 2.
The literal coefficient is x^2y^3.
The variables are x and y.

f. The numerical coefficient is $\frac{1}{2}$.
The literal coefficient is ab.
The variables are a and b.

2. The like terms are grouped as follows:

- m^2n, $2m^2n$, $\frac{1}{2}m^2n$, and $-8m^2n$ are like terms.
- $-5a$, $4a$, and $16a$ are like terms.
- $2mn$ and mn are like terms.
- -9 and 6 are like terms.
- $-3a^2b$ and $-12a^2b$ are like terms.
- $-2ab$ and $-5ab$ are like terms.

3. $6a^{-2}+\frac{4}{a}-2$ is not a polynomial according to the definition. It has a negative exponent and a variable in the denominator. $2a+3b+4c+5d$ is a polynomial, but it has four terms.

4. **a.** monomial **b.** monomial **c.** binomial
d. trinomial **e.** trinomial **f.** trinomial
g. binomial **h.** monomial **i.** trinomial

5. **a.** 3 **b.** 3 **c.** 3
d. 0 **e.** 6 **f.** 4

6. **a.** 2 **b.** 2 **c.** 4
d. 1 **e.** 5 **f.** 3

7. **a.** $3a^2+7a+8$

b. $2b^3+8b^2-4b+6$

c. $-2c+16$

d. $4m^3n^2+6m^2n-2mn-4$

e. $-2x^3y^4+3x^3y^2-x^2y^2+3xy+9$

f. $2x^4y^2z+x^3y^2z-7x^2y^2z^2+4xyz$

8.

Polynomial	Number of Terms (name)	Degree	Number of Variables
$-2+6a+8a^2$	trinomial	2	1
$-3b+7b^3$	binomial	3	1
16	monomial	0	0
$2d$	monomial	1	1
$7y-4xy+2x^2y^3$	trinomial	5	2
$6mn+10m^2n^4$	binomial	6	2
$-1+y-2y^2+3y^3+4y^4$	polynomial	4	1

9. Every polynomial is written with the terms in ascending order.

10. Answers will vary. Some examples have been provided.

a. $5xy^3,\ 2x^2y^2,\ -3x^3y$

b. $4a^2-2a+7,\ a^2+5a-2$

c. $7a+8,\ a+4,\ 2a-9$

d. $6a^5+4a^3+2a^2+a-8,\ 8a^5+2a^4-6a+2$

e. $x^5y-2x^4y^2+3xy,\ 7x^2y^4+2xy-7$

Now Try This

11. Here are four methods of solving this problem.

Method 1: Guessing, Checking, and Revising

If at first you didn't realize that the number of chocolates had to be a multiple of six, you should after your first guess.

Try 30.

$\frac{1}{6}\times 30=5,\ 30-5=25$

$\frac{1}{5}\times 25=5,\ 25-5=20$

$\frac{1}{4}\times 20=5,\ 20-5=15$

$\frac{1}{3}\times 15=5,\ 15-5=10$

$\frac{1}{2}\times 10=5,\ 10-5=5$ ← This is too many.

Try 24.

$\frac{1}{6}\times 24=4,\ 24-4=20$

$\frac{1}{5}\times 20=4,\ 20-4=16$

$\frac{1}{4}\times 16=4,\ 16-4=12$

$\frac{1}{3}\times 12=4,\ 12-4=8$

$\frac{1}{2}\times 8=4,\ 8-4=4$

Therefore, there were 24 chocolates in the box originally.

Method 2: Working Backwards

4 is half of 8. Thus, there were 8 chocolates left before Anthony started eating them.

8 is $\frac{2}{3}$ of what? (Janine had eaten $\frac{1}{3}$.)

$8\times\frac{3}{2}=12$

There were 12 chocolates left when Janine saw the box.

12 is $\frac{3}{4}$ of what? (Gary had eaten $\frac{1}{4}$.)

$12\times\frac{4}{3}=16$

There were 16 chocolates left before Gary got to the box.

16 is $\frac{4}{5}$ of what?

$16 \times \frac{5}{4} = 20$

There were 20 chocolates left when Irene got them.

20 is $\frac{5}{6}$ of what?

$20 \times \frac{6}{5} = 24$

Therefore, there were 24 chocolates in the box originally.

Method 3: Diagram

Mr. Wilkinson ate $\frac{1}{6}$ of the chocolates.

Irene ate $\frac{1}{5}$ of the remaining chocolates.

Gary ate $\frac{1}{4}$ of the remaining chocolates.

Janine ate $\frac{1}{3}$ of the remaining chocolates.

Anthony ate $\frac{1}{2}$ of the remaining chocolates.

4 chocolates are left.

Therefore, there were $4 \times 6 = 24$ chocolates originally.

Method 4: Using Algebra

Let c be the number of chocolates in the box.

$c - \frac{1}{6}c = \frac{5}{6}c$ (number left by Mr. Wilkinson)

$$\frac{5}{6}c - \frac{1}{5}\left(\frac{5}{6}c\right) = \frac{5}{6}c - \frac{1}{6}c$$
$$= \frac{4}{6}c \qquad \text{(number left by Irene)}$$

$$\frac{4}{6}c - \frac{1}{4}\left(\frac{4}{6}c\right) = \frac{4}{6}c - \frac{1}{6}c$$
$$= \frac{3}{6}c \qquad \text{(number left by Gary)}$$

$$\frac{3}{6}c - \frac{1}{3}\left(\frac{3}{6}c\right) = \frac{3}{6}c - \frac{1}{6}c$$

$$= \frac{2}{6}c \qquad \text{(number left by Janine)}$$

$$\frac{2}{6}c - \frac{1}{2}\left(\frac{2}{6}c\right) = \frac{2}{6}c - \frac{1}{6}c$$

$$= \frac{1}{6}c \qquad \text{(number left by Anthony)}$$

$$\therefore \ \frac{1}{6}c = 4$$

$$\frac{1}{\cancel{6}}c \times \cancel{6}^{1} = 4 \times 6$$

$$c = 24$$

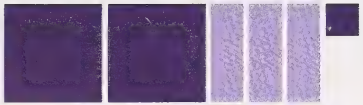
Therefore, there were 24 chocolates originally.

12. The length of an x-tile and the length of a side of an x^2-tile are the same.

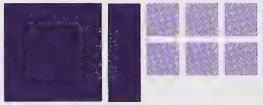
13. The length of a side of a 1-tile and the width of an x-tile are the same.

14. **a.** $2n+5$ **b.** -4 **c.** $x^2 - 3x + 2$
d. $-2m^2 + m - 2$ **e.** $3y^2 + 3y + 4$

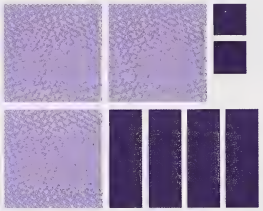
15. **a.**



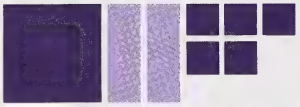
b.



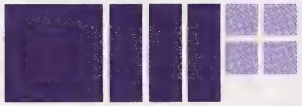
c.



d.



e.



16. **a.** 1 and 5 are congruent; 2 and 6 are congruent; and 3 and 9 are congruent. They are the same size and shape.

b. No, colour does not affect whether or not the shapes are congruent.

17. **a.** 0 **b.** 0 **c.** 0 **d.** 1

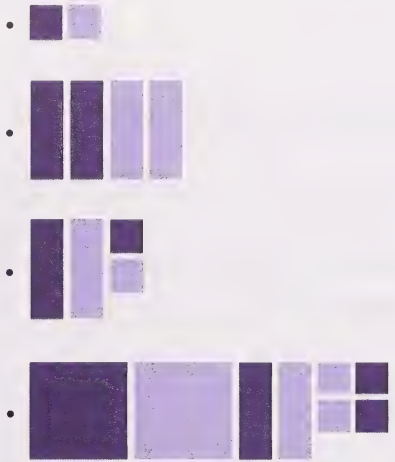
18. **a.** 0 **b.** 0 **c.** 0

19. **a.** $-n^2 - n$ **b.** $2m + 4$

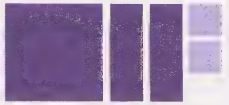
c. $y^2 - y - 2$ **d.** $-2a^2 + 2a + 3$

Note: Other variables may be used.

20. Answers will vary. Here are some examples.

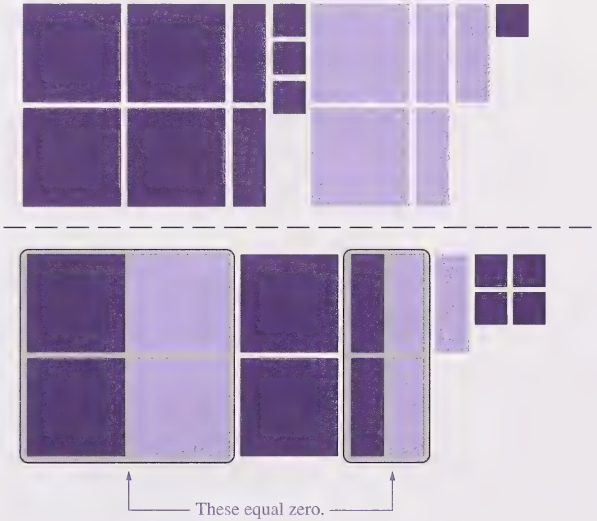


21. **a.** **b.**

c. **d.** 

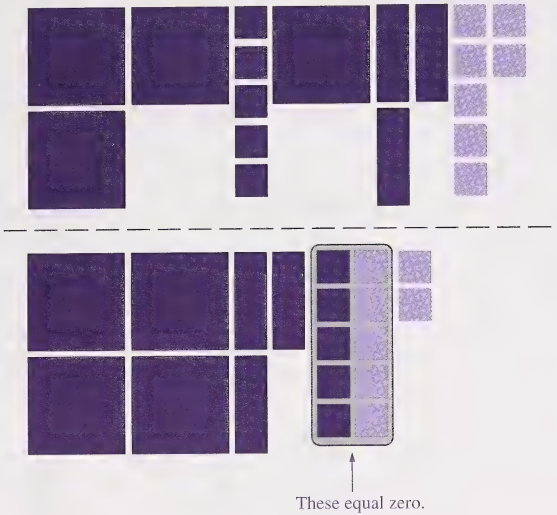
e. **f.**

22. **a.** Using algebra tiles, the answer is illustrated as follows:



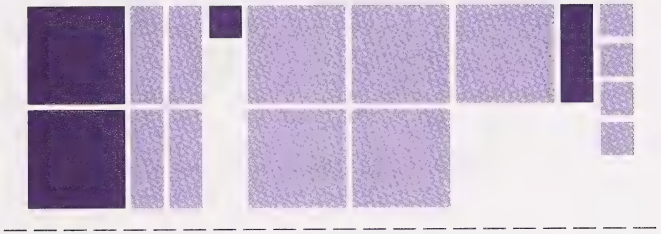
Therefore, the answer is $2a^2 - a + 4$.

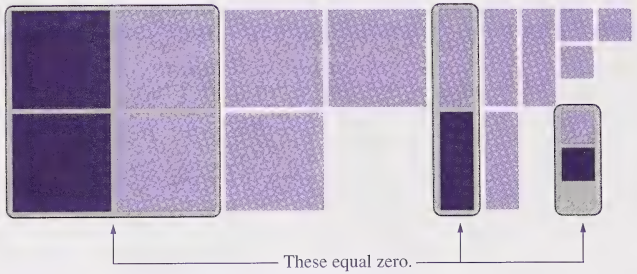
b. Using algebra tiles, the answer is illustrated as follows:



Therefore, the answer is $4b^2 + 3b - 2$.

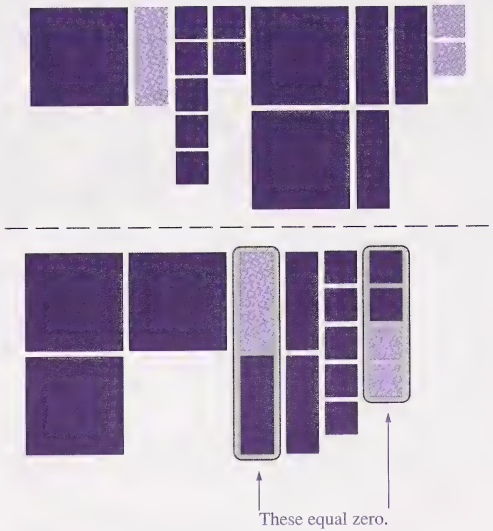
c. Using algebra tiles, the answer is illustrated as follows:





Therefore, the answer is $-3c^2 - 3c - 3$.

d. Using algebra tiles, the answer is illustrated as follows:



Therefore, the answer is $3d^2 + 2d + 5$.

23. To simplify, bring the similar shapes together and eliminate the zeros.

24. **a.** $6m$ **b.** $9n^2$

c. $12y+2$ **d.** $7x^2-3x+1$

e. $-2b^2+b+3$

25. Add the numerical coefficients of the like terms.

26. **a.** $2m+m-m-5=2m-5$

$2+1-1$

b. $3y^2+4y-2y^2+6y-1=3y^2-2y^2+4y+6y-1$

$=y^2+10y-1$

$3-2 \quad 4+6$

c. $x^2-y^2+4x+3y-8+x^2-y^2+2x+3$

$=x^2+x^2-y^2-y^2+4x+2x+3y-8+3$

$=2x^2-2y^2+6x+3y-5$

$1+1 \quad -1-1 \quad 4+2 \quad -8+3$

d. $4a^2b+3a^2+7-2b^2+a^2b+4b^2-3$

$=4a^2b+a^2b+3a^2-2b^2+4b^2+7-3$

$=5a^2b+3a^2+2b^2+4$

$4+1 \quad -2+4 \quad 7-3$

e. $t+t+8-t^2+4t-6+3t^2-2$

$=-t^2+3t^2+t+t+4t+8-6-2$

$=2t^2+6t$

$-1+3 \quad 1+1+4$

27. The answer to the puzzle is **A LOW HA** (aloha).

Now Try This

28. **a.** For this problem, the sum of the three largest three-digit numbers is 2664. Thus, ■ must be a 1 or a 2.

$\therefore$

$$\begin{array}{r} 9\ 9\ 9 \\ 1\ 1\ 1 \\ +\ \blacktriangle\ \blacktriangle\ \blacktriangle \\ \hline \blacksquare\ 9\ 9\ \blacktriangle \end{array}$$

b. For this problem, $8\times$ 5 and $8\times$ 0 are the only combinations to end in zero.

$$\therefore \quad \begin{array}{r} 4\,9\,6\,5 \\ \times \qquad 8 \\ \hline 3\,9\,7\,2\,0 \end{array}$$

29. a. 2 × 3 x^2 − 3 × 3 + 4 =

13.

b. 2 × 3 × 2 +/− + 4 × 2 +/− + 8 =

-12.

c. 4 − 2 × 3 × 2 +/− − 5 × 2 +/− x^2 =

-4.

d. 3 x^2 × 2 +/− − 2 × 3 × 2 +/− x^2 + 3 × 3 × 2 +/− − 4 =

-64.

30. a. 3 × 2 +/− x^2 − 2 × 2 +/− + 5 =

21.

b. 3 × 3 x^2 − 2 × 3 + 5 =

26.

31. a.
$$\begin{aligned} 4a+7-a &= 4a-a+7 \\ &= 3a+7 \\ &= 3(1)+7 \\ &= 3+7 \\ &= 10 \end{aligned}$$

b. $2a^2 - 3b^2 + 2a - 3b^2 = 2a^2 + 2a - 3b^2 - 3b^2$

$= 2a^2 + 2a - 6b^2$

$= 2(1)^2 + 2(1) - 6(2)^2$

$= 2(1) + 2 - 6(4)$

$= 2 + 2 - 24$

$= -20$

c. $8a^2b + 2ab - a^2b + ab + 6 = 8a^2b - a^2b + 2ab + ab + 6$

$= 7a^2b + 3ab + 6$

$= 7(1)^2(2) + 3(1)(2) + 6$

$= 7(1)(2) + 6 + 6$

$= 14 + 6 + 6$

$= 26$

d. $a^3 + b^2 + c + 4a^3 - 3b^2 - 5c + 8$

$= a^3 + 4a^3 + b^2 - 3b^2 + c - 5c + 8$

$= 5a^3 - 2b^2 - 4c + 8$

$= 5(1)^3 - 2(2)^2 - 4(-3) + 8$

$= 5(1) - 2(4) + 12 + 8$

$= 5 - 8 + 12 + 8$

$= 17$

32. The number of points scored by the Flying Eagles is as follows:

$4a + 35b + 17c = 4(3) + 35(2) + 17(1)$

$= 12 + 70 + 17$

$= 99$

The number of points scored by the Leaping Lions is as follows:

$6a + 40b + 10c = 6(3) + 40(2) + 10(1)$

$= 18 + 80 + 10$

$= 108$

Therefore, the Leaping Lions defeated the Flying Eagles 108 to 99.

33. **a.** $3w + 2t + \ell$

b.

Team	w	t	ℓ	Total Points
Victoria	40	3	2	128
Spokane	42	2	3	133
San Diego	27	4	5	94
Portland	47	4	3	152
Seattle	16	3	5	59

c. Portland finished in first place.

d. $72-(47+4+3)=72-54$
$=18$

Portland lost 18 games in regulation time.

34. a. $7x+\square 2\ y-\triangle 2\ z-5y+4z-\bigcirc 3\ x$
$=4x-3y+2z$

b. $\bigcirc 9\ x+2y-3z-\square 5\ y+\triangle 5\ z-5x$
$=4x-3y+2z$

c. $x+y+z+\bigcirc 3\ x-\square 4\ y+\triangle 1\ z$
$=4x-3y+2z$

d. $-3x+\square 4\ y-\triangle 6\ z+\bigcirc 7\ x-7y+8z$
$=4x-3y+2z$

35. Evaluate the original expression with $d=1$ and then evaluate the simplified expression with $d=1$. They should be the same.

Original	Simplified
$4d+8-2d-1+6d-4$	$4d+8-2d-1+6d-4$
$=4(1)+8-2(1)-1+6(1)-4$	$=8d+3$
$=4+8-2-1+6-4$	$=8(1)+3$
$=11$	$=8+3$
	$=11$

36. The answer to the puzzle is **A WRECKED TANGLE** (a rectangle).

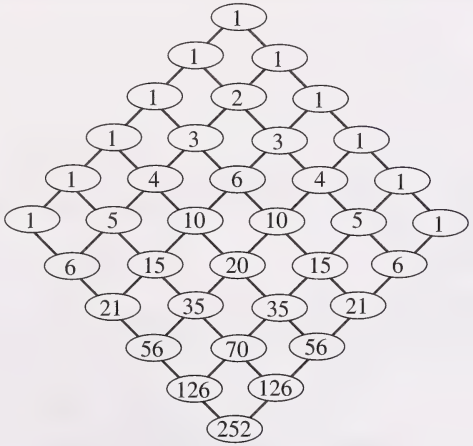
Now Try This

37. One way of solving is by guessing, checking, and revising. You may use as many guesses as required.

	Black Balloons	Red Balloons	White Balloons	Test
Guess 1	5	4	14	$5\times0.65=\$3.25$ $4\times0.25=\$1.00$ $14\times0.15=\$2.10$ $\$6.35$
Guess 2	4	5	14	$4\times0.65=\$2.60$ $5\times0.25=\$1.25$ $14\times0.15=\$2.10$ $\$5.95$
Guess 3	4	6	13	$4\times0.65=\$2.60$ $6\times0.25=\$1.50$ $13\times0.15=\$1.95$ $\$6.05$
Guess 4	4	7	12	$4\times0.65=\$2.60$ $7\times0.25=\$1.75$ $12\times0.15=\$1.80$ $\$6.15$

Barbara bought 4 black, 7 red, and 12 white balloons.

38. There are 252 paths that spell POLYNOMIALS. Study the following model. The number of ways to get to each intersection (letter) is indicated by the circled number.



Section 1: Activity 2

1. a. Tile Diagram A has two x^2-tiles, two x-tiles, and three 1-tiles.

b. Tile Diagram B has one x^2-tile, two x-tiles, and four 1-tiles.

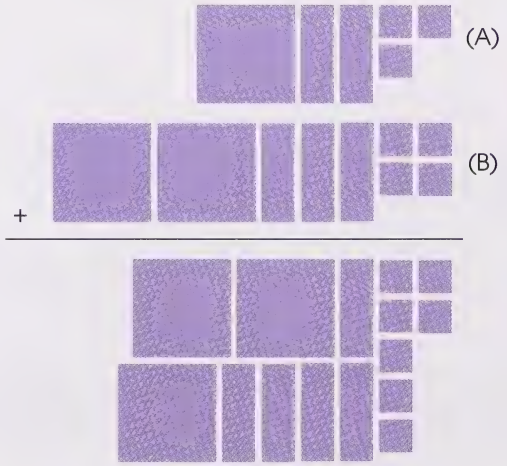
2. Tile Diagrams A and B combined have three x^2-tiles, four x-tiles, and seven 1-tiles.

3. a. $2x^2+2x+3$ **b.** x^2+2x+4 **c.** $3x^2+4x+7$

4. a.

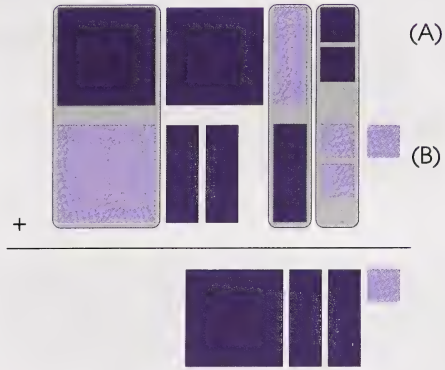
$$\left(x^2+3x+2\right)+\left(x+3\right)=x^2+4x+5$$

b.



$$\left(-x^2-2x-3\right)+\left(-2x^2-3x-4\right)=-3x^2-5x-7$$

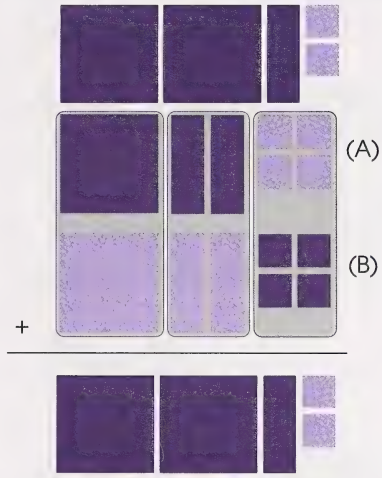
c.



Use the zero principle.

$$(2x^2 - x + 2) + (-x^2 + 3x - 3) = x^2 + 2x - 1$$

d.



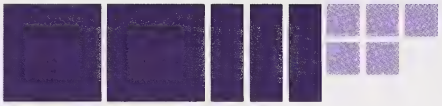
Use the zero principle.

$$(3x^2 + 3x - 6) + (-x^2 - 2x + 4) = 2x^2 + x - 2$$

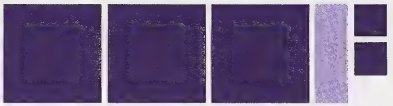
5. You can add the polynomials by adding the numerical coefficients of the like terms.

6. a. Model each expression.

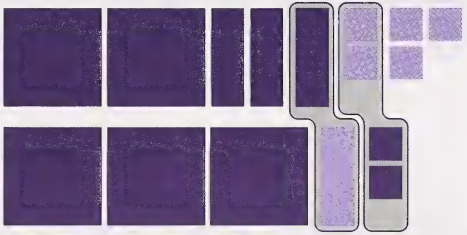
$2x^2 + 3x - 5$



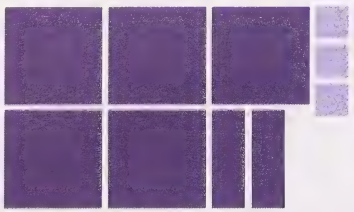
$3x^2 - x + 2$



Use the zero principle to simplify.



Combine like terms.

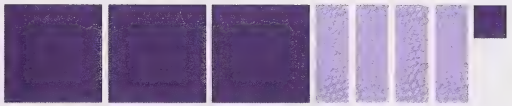


Algebraically, the solution is as follows:

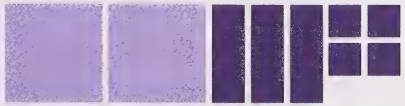
$$\begin{aligned}&\left(2x^2+3x-5\right)+\left(3x^2-x+2\right)\\&=2x^2+3x-5+3x^2-x+2\\&=2x^2+3x^2+3x-x-5+2\\&=5x^2+2x-3\end{aligned}$$

b. Model each expression.

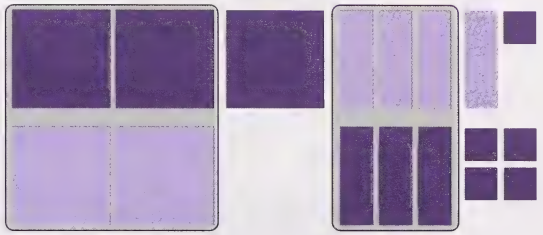
$3y^2-4y+1$



$-2y^2+3y+4$



Use the zero principle to simplify.



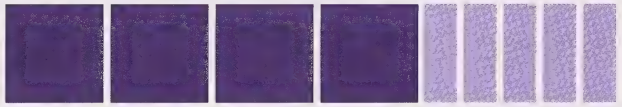
Combine like terms.

Algebraically, the solution is as follows:

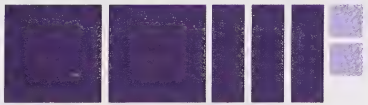
$$\begin{aligned}&\left(3y^2-4y+1\right)+\left(-2y^2+3y+4\right)\\&=3y^2-4y+1-2y^2+3y+4\\&=3y^2-2y^2-4y+3y+1+4\\&=y^2-y+5\end{aligned}$$

c. Model each expression.

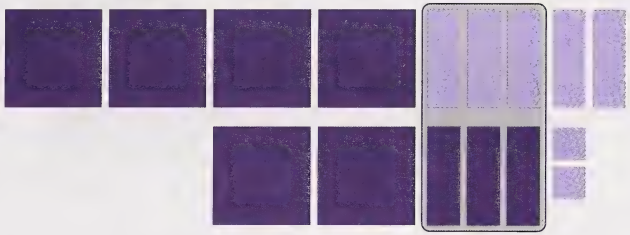
$4a^2-5a$



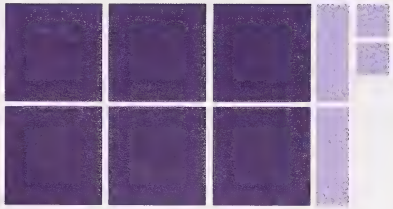
$2a^2 + 3a - 2$



Use the zero principle to simplify.



Combine like terms.

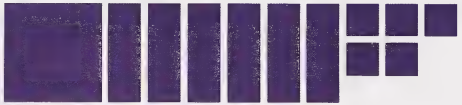


Algebraically, the solution is as follows:

$$\begin{aligned}&\left(4a^2 - 5a\right) + \left(2a^2 + 3a - 2\right)\\ &= 4a^2 - 5a + 2a^2 + 3a - 2\\ &= 4a^2 + 2a^2 - 5a + 3a - 2\\ &= 6a^2 - 2a - 2\end{aligned}$$

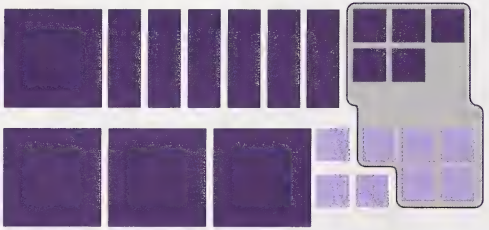
d. Model each expression.

$y^2 + 6y + 5$

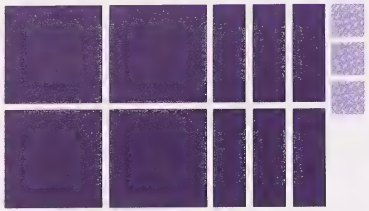


$3y^2 - 8$

Use the zero principle to simplify.



Combine like terms.

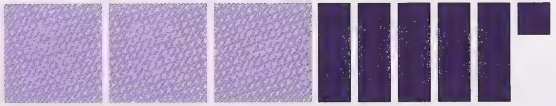


Algebraically, the solution is as follows:

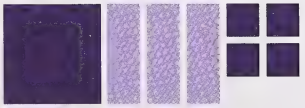
$$\begin{aligned}\left(y^2+6y+5\right)+\left(3y^2-8\right) &= y^2+6y+5+3y^2-8\\ &= y^2+3y^2+6y+5-8\\ &= 4y^2+6y-3\end{aligned}$$

e. Model each expression.

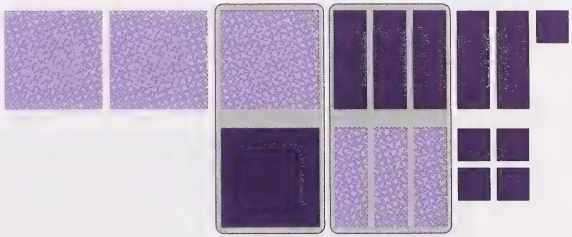
$-3b^2+5b+1$



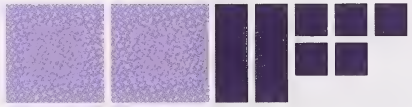
b^2-3b+4



Use the zero principle to simplify.



Combine like terms.



Algebraically, the solution is as follows:

$$\begin{aligned}&\left(-3b^2+5b+1\right)+\left(b^2-3b+4\right)\\ &\quad=-3b^2+5b+1+b^2-3b+4\\ &\quad=-3b^2+b^2+5b-3b+1+4\\ &\quad=-2b^2+2b+5\end{aligned}$$

7. a. $\left(a^2+2a+7\right)+\left(4a^2-5a+3\right)$

$$\begin{aligned}&=a^2+2a+7+4a^2-5a+3\\ &=a^2+4a^2+2a-5a+7+3\\ &=5a^2-3a+10\end{aligned}$$

b. $(3b^2-9b+1)+(-5b^2+2b+3)$

$=3b^2-9b+1-5b^2+2b+3$

$=3b^2-5b^2-9b+2b+1+3$

$=-2b^2-7b+4$

c. $(4c^4+5c^2+7c+1)+(3c^4+2c^2-6c-4)$

$=4c^4+5c^2+7c+1+3c^4+2c^2-6c-4$

$=4c^4+3c^4+5c^2+2c^2+7c-6c+1-4$

$=7c^4+7c^2+c-3$

d. $(d^3+d^2-4+3d)+(2d^3-5d+6d^2)$

$=d^3+d^2-4+3d+2d^3-5d+6d^2$

$=d^3+2d^3+d^2+6d^2+3d-5d-4$

$=3d^3+7d^2-2d-4$

e. $(m^2+7m-3)+(2m^2-5)+(-5m^2+m+1)$

$=m^2+7m-3+2m^2-5-5m^2+m+1$

$=m^2+2m^2-5m^2+7m+m-3-5+1$

$=-2m^2+8m-7$

8. a.

$$\begin{array}{r} 2x^2+7 \\ +\ (3x^2-4) \\ \hline 5x^2+3 \end{array}$$

b.

$$\begin{array}{r} 3a^2+5a+6 \\ +\ (-4a^2+2a-3) \\ \hline -a^2+7a+3 \end{array}$$

c.

$$\begin{array}{r} 8n^2-5n+7 \\ +\ (-3n^2\qquad -4) \\ \hline 5n^2-5n+3 \end{array}$$

d.

$$\begin{array}{r} 3m^3+2m^2-4m+8 \\ +\ (\ m^3\qquad +7m-6) \\ \hline 4m^3+2m^2+3m+2 \end{array}$$

9. a. $(2x+3)+(7x)+(5x+1)$

b. $(2x+3)+(7x)+(5x+1)=2x+3+7x+5x+1$

$=2x+7x+5x+3+1$

$=14x+4$

c. $14x+4=14(3)+4$

$=42+4$

$=46$

The perimeter is 46 cm.

10. a. $(2x+3)+(3x-2)+(x+8) = 2x+3+3x-2+x+8$
$= 2x+3x+x+3-2+8$
$= 6x+9$

b. $6x+9 = 6(5)+9$
$= 30+9$
$= 39$

The perimeter of the plot is 39 m.

c. The plot of land is in the shape of an equilateral triangle since each side has the same length. You can check this by substituting 5 for x in each of the three parts of the expression.

11. a. $\left(4x^2+3x-2\right)+\left(x^2-x+5\right)$
$= 4x^2+3x-2+x^2-x+5$
$= 4x^2+x^2+3x-x-2+5$
$= 5x^2+2x+3$

b. Substitute $x = 2$ into the original expression.

$\left(4x^2+3x-2\right)+\left(x^2-x+5\right)$
$= \left[4(2)^2+3(2)-2\right]+\left[(2)^2-2+5\right]$
$= \left[4(4)+3(2)-2\right]+(4-2+5)$
$= (16+6-2)+7$
$= 20+7$
$= 27$

Substitute $x = 2$ into the answer to question 11.a.

$5x^2+2x+3 = 5(2)^2+2(2)+3$
$= 5(4)+2(2)+3$
$= 20+4+3$
$= 27$

The value for the original expression equals the value for the answer to question 11.a. So, the original expression is equivalent to the answer to question 11.a.

12. a. Tile Diagram A has three x^2-tiles, four x-tiles, and five 1-tiles.

b. Tile Diagram B has two x^2-tiles, one x-tile, and three 1-tiles.

13. One x^2-tile, three x-tiles, and two 1-tiles are left.

14. **a.** $3x^2+4x+5$ **b.** $2x^2+x+3$

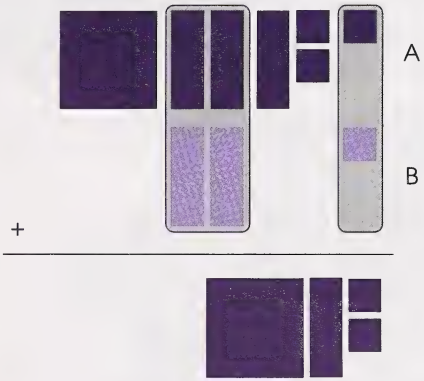
c. x^2+3x+2

15. **a.** x^2+3x+2

b. The answers are the same.

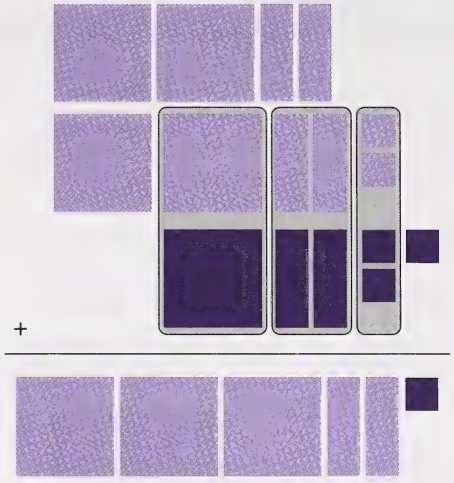
c. You can subtract polynomials by adding the opposite coefficient of each like term.

16. **a.**



$(x^2+3x+3)-(2x+1)=x^2+x+2$

b.

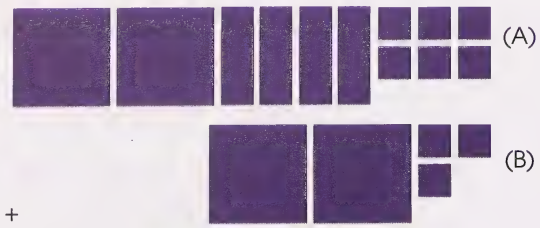


$(-4x^2-4x-2)-(-x^2-2x-3)=-3x^2-2x+1$

c.

$(-2x^2-3x-3)-(x^2+2x+3)=-3x^2-5x-6$

d.

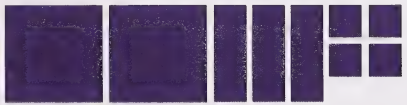


$$\left(2x^2+4x+6\right)-\left(-2x^2-3\right)=4x^2+4x+9$$

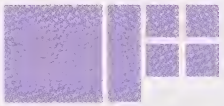
17. You take the opposite of each numerical coefficient in Tile Diagram B and add it to the numerical coefficient of each like term in Tile Diagram A.

18. **a.** Model each expression using the opposite value of each tile in the second polynomial.

$2x^2+3x+4$

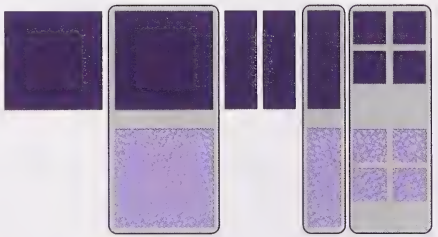


$-x^2-x-4$

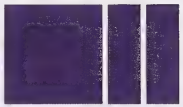


The opposite value of x^2+x+4 is $-x^2-x-4$.

Use the zero principle to simplify.



Combine like terms.

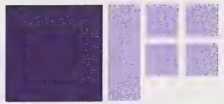


Algebraically, the solution is as follows:

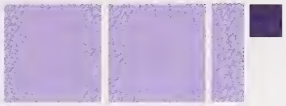
$$\begin{aligned}&\left(2x^2+3x+4\right)-\left(x^2+x+4\right)\\&=\left(2x^2+3x+4\right)+\left(-x^2-x-4\right)\\&=2x^2+3x+4-x^2-x-4\\&=2x^2-x^2+3x-x+4-4\\&=x^2+2x\end{aligned}$$

b. Model each expression using the opposite value of each tile in the polynomial you are subtracting.

$x^2 - x - 4$

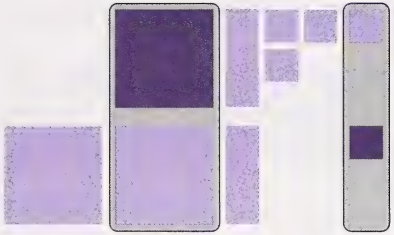


$-2x^2 - x + 1$

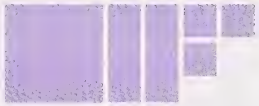


The opposite value of $2x^2 + x - 1$ is $-2x^2 - x + 1$.

Use the zero principle to simplify.



Combine like terms.

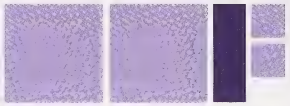


Algebraically, the solution is as follows:

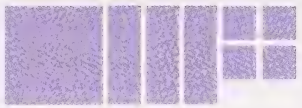
$$\begin{aligned}
&\left(x^2 - x - 4\right) - \left(2x^2 + x - 1\right) \\
&= \left(x^2 - x - 4\right) + \left(-2x^2 - x + 1\right) \\
&= x^2 - x - 4 - 2x^2 - x + 1 \\
&= x^2 - 2x^2 - x - x - 4 + 1 \\
&= -x^2 - 2x - 3
\end{aligned}$$

c. Model each expression using the opposite value of each tile in the polynomial you are subtracting.

$-2x^2 + x - 2$

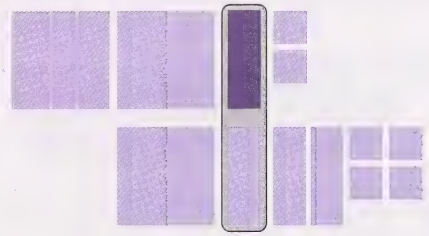


$-x^2 - 3x - 4$

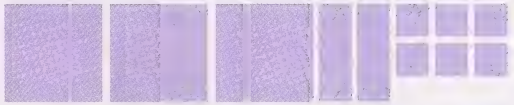


The opposite value of $x^2 + 3x + 4$ is $-x^2 - 3x - 4$.

Use the zero principle to simplify.



Combine like terms.



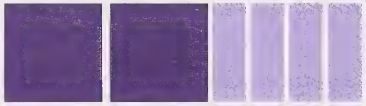
Algebraically, the solution is as follows:

$$\begin{aligned}&\left(-2x^2+x-2\right)-\left(x^2+3x+4\right)\\&=\left(-2x^2+x-2\right)+\left(-x^2-3x-4\right)\\&=-2x^2+x-2-x^2-3x-4\\&=-2x^2-x^2+x-3x-2-4\\&=-3x^2-2x-6\end{aligned}$$

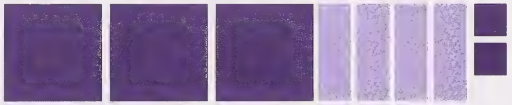
d. Model each expression using the opposite value of each tile in the second polynomial.

x^2+2

$2x^2-4x$



Combine like terms.



There are no zero values.

Algebraically, the solution is as follows:

$$\begin{aligned}\left(x^2+2\right)-\left(-2x^2+4x\right)&=\left(x^2+2\right)+\left(2x^2-4x\right)\\&=x^2+2+2x^2-4x\\&=x^2+2x^2-4x+2\\&=3x^2-4x+2\end{aligned}$$

19. a. $(4y^2+3y+5)-(y^2+3y+2)$

$=(4y^2+3y+5)+(-y^2-3y-2)$

$=4y^2+3y+5-y^2-3y-2$

$=4y^2-y^2+3y-3y+5-2$

$=3y^2+3$

b. $(-3y^2-y+7)-(2y^2-3y+3)$

$=(-3y^2-y+7)+(-2y^2+3y-3)$

$=-3y^2-y+7-2y^2+3y-3$

$=-3y^2-2y^2-y+3y+7-3$

$=-5y^2+2y+4$

c. $(3y^3+3y^2-2y+5)-(y^3-2y^2+4y-2)$

$=(3y^3+3y^2-2y+5)+(-y^3+2y^2-4y+2)$

$=3y^3+3y^2-2y+5-y^3+2y^2-4y+2$

$=3y^3-y^3+3y^2+2y^2-2y-4y+5+2$

$=2y^3+5y^2-6y+7$

d. $(y^2-4y+5)-(y^2+6)=(y^2-4y+5)+(-y^2-6)$

$=y^2-4y+5-y^2-6$

$=y^2-y^2-4y+5-6$

$=-4y-1$

20. a.

$$\begin{array}{r} 4a^2+3a+5 \\ -\left(2a^2+2a+3\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 4a^2+3a+5 \\ +\left(-2a^2-2a-3\right) \\ \hline 2a^2+a+2 \end{array}$$

b.

$$\begin{array}{r} 2b^2+7b-3 \\ -\left(b^2-2b-1\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 2b^2+7b-3 \\ +\left(-b^2+2b+1\right) \\ \hline b^2+9b-2 \end{array}$$

c.

$$\begin{array}{r} -6c^2+5c-8 \\ -\left(4c^2\quad +7\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} -6c^2+5c-8 \\ +\left(-4c^2\quad -7\right) \\ \hline -10c^2+5c-15 \end{array}$$

d.

$$\begin{array}{r} 8n^3+4n^2-2n+7 \\ -\left(2n^3-3n^2+n+2\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 8n^3+4n^2-2n+7 \\ +\left(-2n^3+3n^2-n-2\right) \\ \hline 6n^3+7n^2-3n+5 \end{array}$$

21. a. $(4x^2+4x+1)-(x^2+2x-3)$

b. $(4x^2+4x+1)-(x^2+2x-3)$

$=(4x^2+4x+1)+(-x^2-2x+3)$

$=4x^2+4x+1-x^2-2x+3$

$=4x^2-x^2+4x-2x+1+3$

$=3x^2+2x+4$

c. $3x^2+2x+4=3(4)^2+2(4)+4$

$=3(16)+8+4$

$=48+8+4$

$=60$

The area of the shaded area is $60\,\text{cm}^2$.

22. The answer to the puzzle is as follows:

Romantic: **AN ITALIAN INSECT**
American: **A HAPPY CONTAINER**

23. a. Area of large circle $=\pi(x)^2$

$=\pi x^2$

Area of one small circle $=\pi\left(\frac{x}{2}\right)^2$

$=\frac{\pi x^2}{4}$

Area of two small circles $=\frac{1}{2}\left(\frac{\pi x^2}{4}\right)$

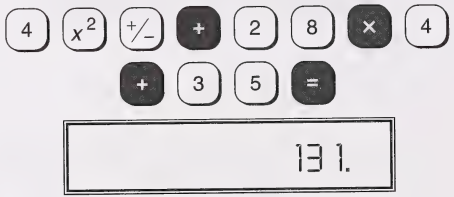
$=\frac{\pi x^2}{2}$

$\therefore$ Area of shaded region $=\pi x^2-\frac{\pi x^2}{2}$

b. $\pi x^2-\frac{\pi x^2}{2}=\frac{2\pi x^2}{2}-\frac{\pi x^2}{2}$

$=\frac{\pi x^2}{2}$

The area of the shaded region is the same as the area of the two small circles.

24. a. 

The temperature after 4 min is 131°C.

b. 9 x^2 +/− + 2 8 × 9 + 3 5 =

The temperature after 9 min is 206°C.

c. 1 2 x^2 +/− + 2 8 × 1 2 + 3 5 =

The maximum temperature the oven operates at is 227°C.

Now Try This

25. a.

i.	30	ii.	18	iii.	23
iv.	14	v.	19	vi.	9
vii.	7	viii.	8	ix.	5
x.	3	xi.	4		

b. 1

26. No, the primes obtained for each polynomial using the same value of n are different.

27. If $n = 20$, both polynomials produce the value of 421.

28. Yes, 421 is a prime number.

29. For $n < 20$, the values produced by Euler's polynomial are less than those produced by Escott's polynomial. For $n > 20$, the values produced by Euler's polynomial are greater than those produced by Escott's polynomial.

30. No, the value of 1681 obtained from Euler's polynomial is not a prime number.

31. No, the value 1681 obtained from Escott's polynomial is not a prime number.

32. Euler's polynomial will give values that are prime when $n \leq 40$. Escott's polynomial will give values that are prime for $n \leq 79$.

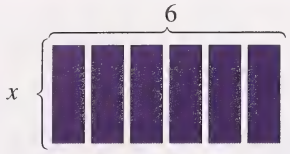
33. Some are prime numbers and some are not.

34. The polynomials can produce prime numbers for n up to a certain number, but values of n greater than a certain number do not necessarily give prime numbers.

Section 1: Activity 3

1. a. Since there are six rectangles, each having an area of x square units, the area is $6x$ square units.

b. The group of tiles can be labelled as follows:

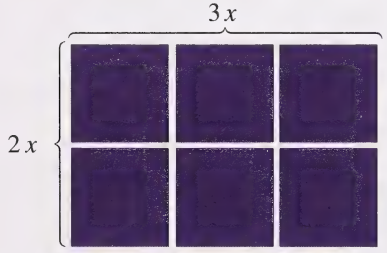


Therefore, the length is 6 units and the width is x units.

c. $6(x) = 6x$ square units

2. a. Since there are six squares, each having an area of x^2 square units, the area is $6x^2$ square units.

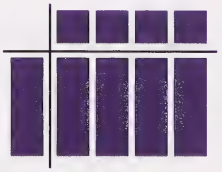
b. The group of tiles can be labelled as follows:



Therefore, the length is $3x$ units and the width is $2x$ units.

c. $(3x)(2x) = 6x^2$ square units

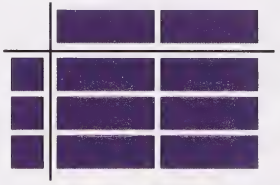
3. a.



4 is represented horizontally, and x is represented vertically.

The area is $4x$ square units.

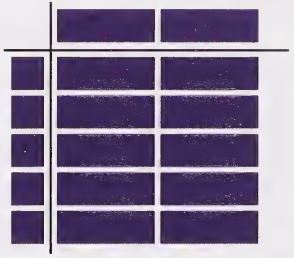
b.

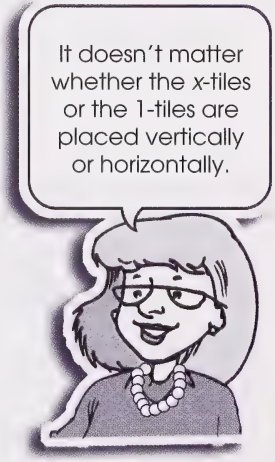


$2x$ is represented horizontally, and 3 is represented vertically.

The area is $6x$ square units.

c.





$2x$ is represented horizontally, and 5 is represented vertically.

The area is $10x$ square units.

4. A method you can use to find the product of two monomials without using algebra tiles is to multiply the numerical coefficients and multiply the variables.

5. a. $(2x)(4x) = (2)(4)(x)(x)$
$= 8x^2$

b. $(3x)(5x) = (3)(5)(x)(x)$
$= 15x^2$

c. $(5)(2x) = (5)(2)(x)$
$= 10x$

d. $(x)(4) = (1)(4)(x)$
$= 4x$

e. $(3)(6x) = (3)(6)(x)$
$= 18x$

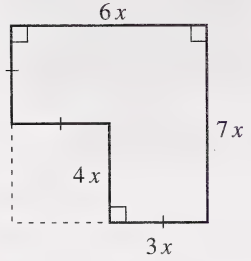
f. $(2x)(3x) = (2)(3)(x)(x)$
$= 6x^2$

6. a. $4mn$ **b.** $12a^4$ **c.** $18xy^2$
d. $-8x^3y^2$ **e.** $10a^2b^2$ **f.** $12x^2yz$
g. $-8ab^2$ **h.** $12x^2y^5$ **i.** $6a^2bc$
j. $12a^3b^5c^2$ **k.** $24m^4$ **l.** $-6a^6b^3$
m. $8a^3$ **n.** $8a^3$

7. a. $(2x)(3y) = 6xy$

b. $(3y)^2 = 9y^2$

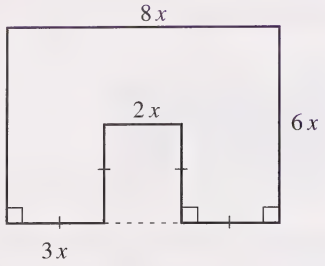
c.



$(6x)(7x) - (3x)(4x) = 42x^2 - 12x^2$
$= 30x^2$

d. $(7ab)(4) = 28ab$

e.

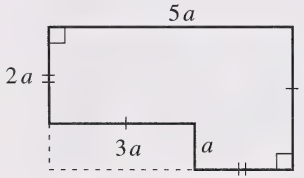


$$(8x)(6x)-(3x)(2x)=48x^2-6x^2$$
$$=42x^2$$

8. a. $P=2\ell+2w$
$=2(5m)+2(2m)$
$=10m+4m$
$=14m$

$A=\ell\times w$
$=(5m)(2m)$
$=10m^2$

b.



$P=5a+3a+2a+a+3a+2a$
$=16a$

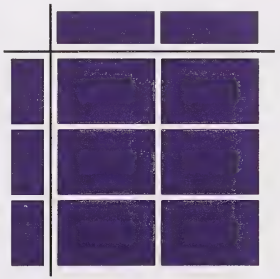
$A=(5a)(3a)-(3a)(a)$
$=15a^2-3a^2$
$=12a^2$

9. Answers will vary. Some examples are provided.

- $(3a)(2b)$
- $(2a)(3b)$
- $(2ab)(3)$
- $(3ab)(2)$
- $(a)(6b)$
- $(6a)(b)$
- $(ab)(6)$
- $(6ab)(1)$

10. The product of $2x^2$ and $3x$ results in a monomial of degree three. Since x^2 is a square, a good choice for x^3 would be a cube. The product of $2x^2$ and $3x$ is $6x^3$ and could be modelled with six cubes.

11. Answers will vary. $(2x)(3y)=6xy$ can be modelled as follows:



Henrietta can use the regular x-tiles for $2x$. But, she will have to choose a different size tiles for $3y$ since the y and xy variables are different from x, x^2, and 1.

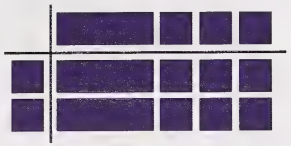
Section 1: Activity 4

1. To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial.

2. a. $2x+6$ **b.** x^2+3x

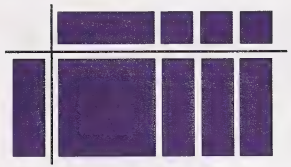
c. $2x^2+5x$ **d.** $3x+3$

3. a.



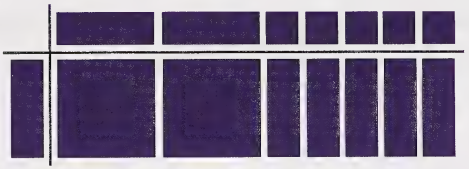
Expressed as a product of its dimensions, the area is $2(x+3)$.

b.



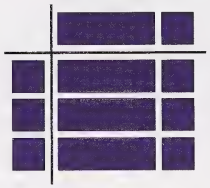
Expressed as a product of its dimensions, the area is $x(x+3)$.

c.



Expressed as a product of its dimensions, the area is $x(2x+5)$.

d.



Expressed as a product of its dimensions, the area is $3(x+1)$.

4. a. $2(x+3)=2(x)+2(3)$
$=2x+6$

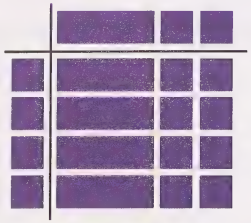
b. $x(x+3)=x(x)+x(3)$
$=x^2+3x$

c. $x(2x+5)=x(2x)+x(5)$
$=2x^2+5x$

d. $3(x+1)=3(x)+3(1)$
$=3x+3$

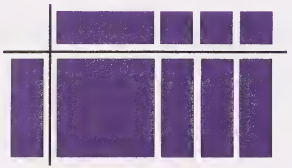
The answers here are the same as the answers to question 2.

5. a.



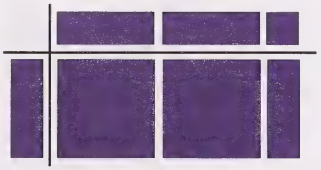
The area is $4x+8$.

b.



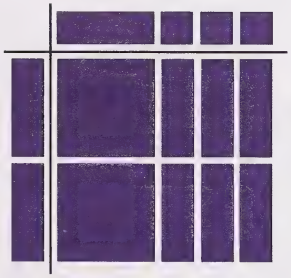
The area is x^2+3x.

c.



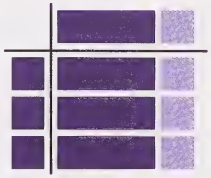
The area is $2x^2+x$.

d.



The area is $2x^2+6x$.

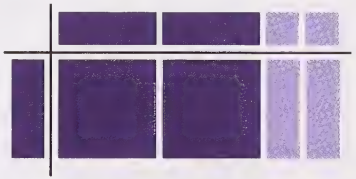
6. a.



The area, expressed as a product of its dimensions, is $3(x-1)$.

The area, expressed as a single polynomial, is $3x-3$.

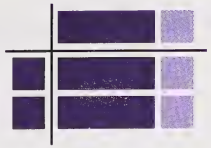
b.



The area, expressed as a product of its dimensions, is $x(2x-2)$.

The area, expressed as a single polynomial, is $2x^2-2x$.

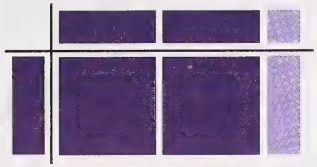
c.



The area, expressed as a product of its dimensions, is $2(x-1)$.

The area, expressed as a single polynomial, is $2x-2$.

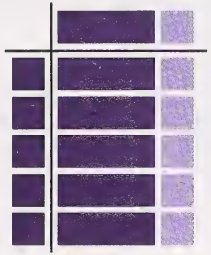
d.



The area, expressed as a product of its dimensions, is $x(2x-1)$.

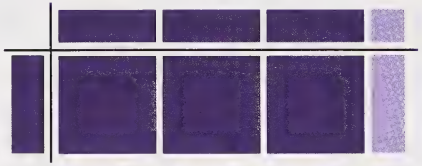
The area, expressed as a single polynomial, is $2x^2-x$.

7. a.



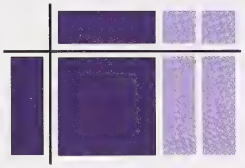
The area is $5x-5$.

b.



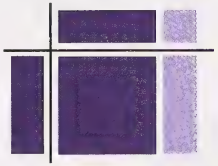
The area is $3x^2-x$.

c.



The area is x^2-2x.

d.



The area is x^2-x.

8. a. $4x-4$ **b.** a^2+4a

c. $5x^2+5x+5$ **d.** $2b^2-6b$

e. $2a^2+3a$ **f.** $2y^3-2y^2+6y$

9. a. $2n^3+6n$

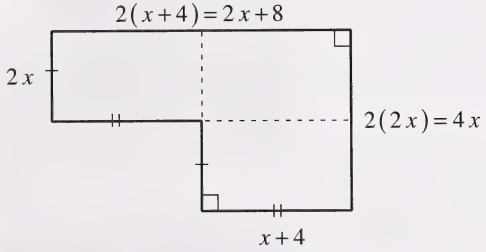
b. $3d^3+6d^2-12d$

c. $2a^2+4a+6+3a^2-3a+3=5a^2+a+9$

d. $-3a^3-3a^2+12a$

10. a. $3x(x+3)=3x^2+9x$

b.



Method 1

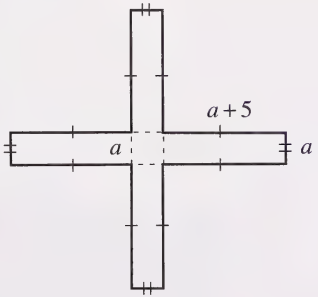
$$2x(x+4)+4x(x+4)=2x^2+8x+4x^2+16x$$
$$=6x^2+24x$$

Method 2

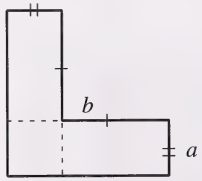
$$2x(2x+8)+2x(x+4)=4x^2+16x+2x^2+8x$$
$$=6x^2+24x$$

c. $4[a(a+5)]+(a\times a)=4\left(a^2+5a\right)+a^2$

$$=4a^2+20a+a^2$$
$$=5a^2+20a$$

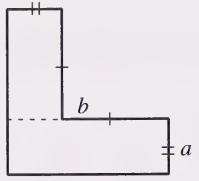


11. Method 1



$$a(a)+a(b)+a(b)$$
$$=a^2+ab+ab$$
$$=a^2+2ab$$

Method 2



$$a(b)+a(a+b)$$
$$=ab+a^2+ab$$
$$=a^2+2ab$$

12. $(4x)(4x)(4x) = (4)(4)(4)(x)(x)(x)$

$= 64x^3$

The volume is $64x^3$ units.

13. $a(a+1) = a^2 + a$

14. Square the smallest number; then add it to the smallest number.

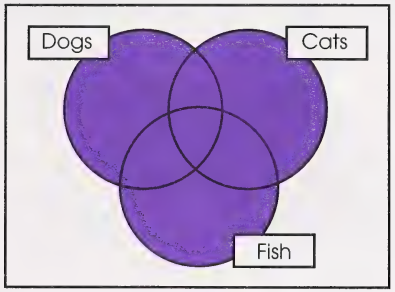
15. $(-4) \times (-3) = 12$ $\qquad (-4)^2 + (-4) = 16 + (-4)$

$= 12$

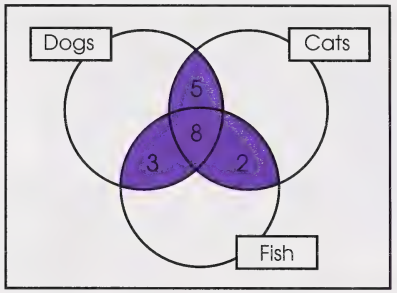
Yes, the rule applies to negative integers as well.

16. You can use a Venn diagram to solve this problem.

Step 1: There are three kinds of pets. So, three interlocking circles are used.



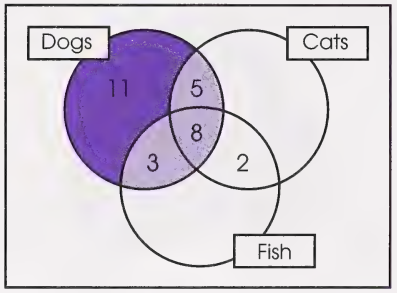
Step 2: There are five families that have only cats and dogs, three families that have only dogs and fish, two families that have only cats and fish, and eight families that have all three.



Step 3: Calculate the number of families that have only dogs.

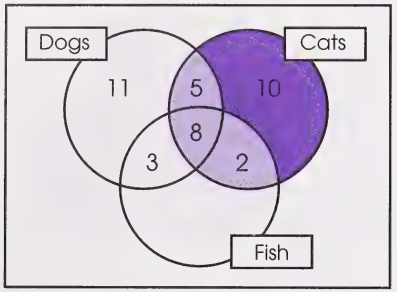
$27 - (5 + 8 + 3) = 27 - 16$

$= 11$



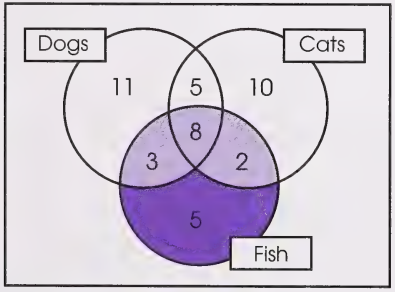
Step 4: Calculate the number of families that have only cats.

$$25-(5+8+2)=25-15$$
$$=10$$



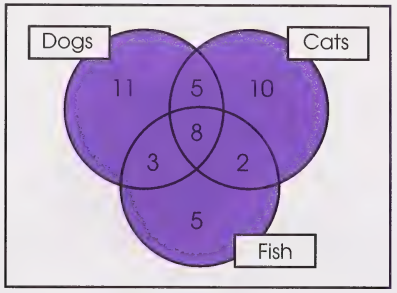
Step 5: Calculate the number of families that have only fish.

$$18-(3+8+2)=18-13$$
$$=5$$



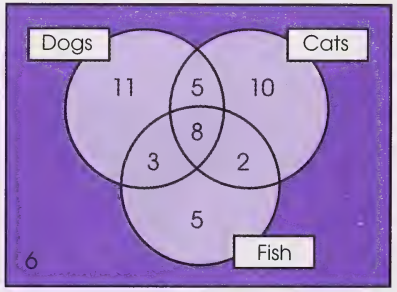
Step 6: Calculate the number of families that have either dogs, cats, or fish.

$$5+8+3+2+11+10+5=44$$



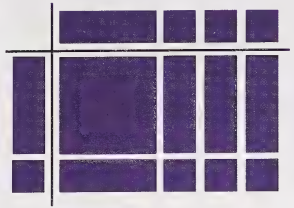
Step 7: Calculate the number of families that do not have dogs, cats, or fish.

$$50-44=6$$



Section 1: Activity 5

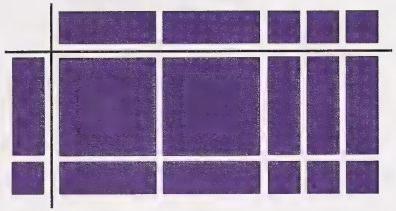
1. a.



The area is $x^2 + 4x + 3$.

Therefore, $(x+3)(x+1) = x^2 + 4x + 3$.

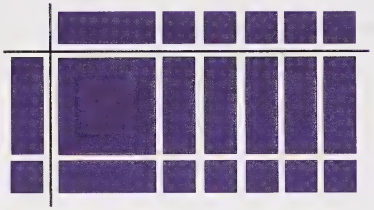
b.



The area is $2x^2 + 5x + 3$.

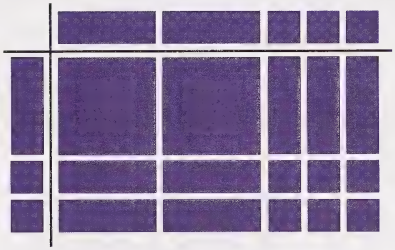
Therefore, $(2x+3)(x+1) = 2x^2 + 5x + 3$.

2. a.



The area is $(x+5)(x+1)$.

b.



The area is $(2x+3)(x+2)$.

3. Multiply each term of the first binomial by each term of the second binomial; then collect the like terms.

4. a.
$$\begin{aligned}(x+1)(x+6) &= (x)(x)+(x)(6)+(1)(x)+(1)(6)\\ &= x^2 + 6x + x + 6\\ &= x^2 + 7x + 6\end{aligned}$$

b. $(x+6)(x+3)=(x)(x)+(x)(3)+(6)(x)+(6)(3)$

$=x^2+3x+6x+18$

$=x^2+9x+18$

c. $(x+4)(x+3)=(x)(x)+(x)(3)+(4)(x)+(4)(3)$

$=x^2+3x+4x+12$

$=x^2+7x+12$

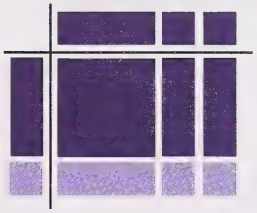
d. $(2x+3)(4x+2)$

$=(2x)(4x)+(2x)(2)+(3)(4x)+(3)(2)$

$=8x^2+4x+12x+6$

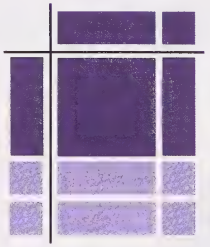
$=8x^2+16x+6$

5. a.



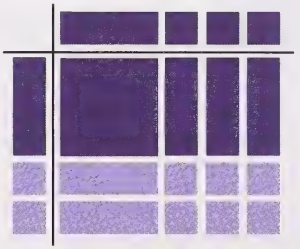
$(x+2)(x-1)=x^2+x-2$

b.



$(x+1)(x-2)=x^2-x-2$

6. a.



The area, expressed as a product of its dimensions, is $(x+3)(x-2)$.

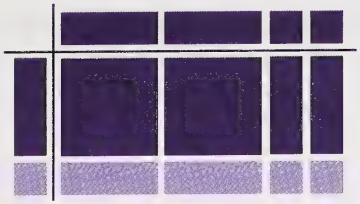
$(x+3)(x-2)=(x)(x)+(x)(-2)+(3)(x)+(3)(-2)$

$=x^2-2x+3x-6$

$=x^2+x-6$

The area is x^2+x-6.

b.



The area, expressed as a product of its dimensions, is $(2x+2)(x-1)$.

$$\begin{aligned}(2x+2)(x-1)&=(2x)(x)+(2x)(-1)+(2)(x)+(2)(-1)\\&=2x^2-2x+2x-2\\&=2x^2-2\end{aligned}$$

The area is $2x^2-2$.

7. a. $$\begin{aligned}(x+4)(x+3)&=(x)(x)+(x)(3)+(4)(x)+(4)(3)\\&=x^2+3x+4x+12\\&=x^2+7x+12\end{aligned}$$

b. $$\begin{aligned}(x-2)(x+5)&=(x)(x)+(x)(5)+(-2)(x)+(-2)(5)\\&=x^2+5x-2x-10\\&=x^2+3x-10\end{aligned}$$

c. $$\begin{aligned}(2x-3)(x+4)&=(2x)(x)+(2x)(4)+(-3)(x)+(-3)(4)\\&=2x^2+8x-3x-12\\&=2x^2+5x-12\end{aligned}$$

d. $$\begin{aligned}(x+6)(x-1)&=(x)(x)+(x)(-1)+(6)(x)+(6)(-1)\\&=x^2-x+6x-6\\&=x^2+5x-6\end{aligned}$$

e. $$\begin{aligned}(2x+6)(x+2)&=(2x)(x)+(2x)(2)+(6)(x)+(6)(2)\\&=2x^2+4x+6x+12\\&=2x^2+10x+12\end{aligned}$$

f. $$\begin{aligned}(x-1)(x-1)&=(x)(x)+(x)(-1)+(-1)(x)+(-1)(-1)\\&=x^2-x-x+1\\&=x^2-2x+1\end{aligned}$$

8. a. n^2 means "n times n."

b. $$\begin{aligned}(x+2)(x+2)&=(x)(x)+(x)(2)+(2)(x)+(2)(2)\\&=x^2+2x+2x+4\\&=x^2+4x+4\end{aligned}$$

c. $(x+3)(x+3)=(x)(x)+(x)(3)+(3)(x)+(3)(3)$

$=x^2+3x+3x+9$

$=x^2+6x+9$

d. $(x+4)^2=x^2+4x+4x+16$

$=x^2+8x+16$

Therefore, it is a perfect square trinomial.

9. a. $(x-4)(x+4)=x^2+4x-4x-16$

$=x^2-16$

b. $(a+3)(a-3)=a^2-3a+3a-9$

$=a^2-9$

c. $(2x-1)(2x+1)=4x^2+2x-2x-1$

$=4x^2-1$

d. $(y-2)(y+2)=y^2+2y-2y-4$

$=y^2-4$

10. a. The similarities are as follows:

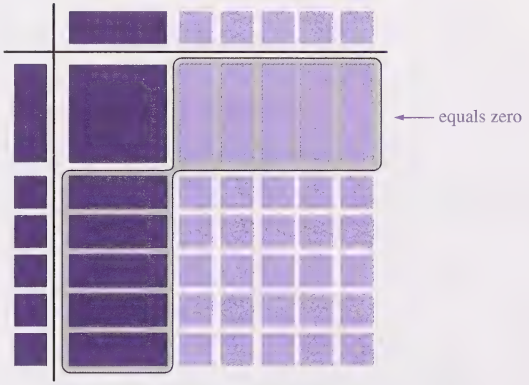
- There are only two terms in each answer.
- The product is always a difference of those two terms.

b. They are both perfect squares.

c. Yes, $(x-5)(x+5)=x^2-25$

Both terms are perfect squares.

d.



The equal numbers of x-tiles and $-x$-tiles add up to zero.

Therefore, you are left with x^2-25.

11. a. $(2x+3)(2x-3)$

$=(2x)(2x)+(2x)(-3)+(3)(2x)+(3)(-3)$

$=4x^2-6x+6x-9$

$=4x^2-9$

b. $(x+4)(x-1)=(x)(x)+(x)(-1)+(4)(x)+(4)(-1)$

$=x^2-x+4x-4$

$=x^2+3x-4$

c. $(x+3)(x-5)=(x)(x)+(x)(-5)+(3)(x)+(3)(-5)$

$=x^2-5x+3x-15$

$=x^2-2x-15$

d. $(x+7)^2=(x+7)(x+7)$

$=(x)(x)+(x)(7)+(7)(x)+(7)(7)$

$=x^2+7x+7x+49$

$=x^2+14x+49$

e. $(2x+5)(x+2)$

$=(2x)(x)+(2x)(2)+(5)(x)+(5)(2)$

$=2x^2+4x+5x+10$

$=2x^2+9x+10$

f. $(x-9)(x-6)$

$=(x)(x)+(x)(-6)+(-9)(x)+(-9)(-6)$

$=x^2-6x-9x+54$

$=x^2-15x+54$

12. a. $P=2(x+2)+2(x+6)$

$=2x+4+2x+12$

$=4x+16$ units

$A=(x+6)(x+2)$

$=x^2+2x+6x+12$

$=x^2+8x+12$ square units

b. $P=4(x+3)$

$=4x+12$ units

$A=(x+3)^2$

$=(x+3)(x+3)$

$=x^2+3x+3x+9$

$=x^2+6x+9$ square units

13. The answer to the puzzle is **IT'S A CHEW** (achoo).

Now Try This

14. Method 1: Algebraically

Let n be the smallest number. Thus, $n+1$ is the larger number.

$(n+1)^2-n^2=81$

$n^2+2n+1-n^2=81$

$2n+1=81$

$2n=80$

$n=40$

The numbers are 40 and 41.

Method 2: Guessing, Checking, and Revising

$30^2 - 29^2 = 900 - 841 = 59$ (too low)

$50^2 - 49^2 = 2500 - 2401 = 99$ (too high)

$40^2 - 39^2 = 1600 - 1521 = 79$ (too low, but close)

$41^2 - 40^2 = 1681 - 1600 = 81$

Method 3: Patterns

1 4 9 16 25 36 49

- The difference between the first and second perfect squares is 3.
- The difference between the second and third perfect squares is 5.
- The difference between the third and fourth perfect squares is 7.

Note: $1 + 2 = 3$

$2 + 3 = 5$

$3 + 4 = 7$

Thus, the difference between the fortieth and forty-first perfect squares would be 81.

15. Make an organized list of the possible combinations of Marten's placements.

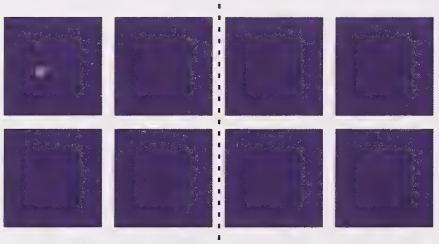
First	Second	Third
3	0	0
2	1	1
1	2	2
1	0	5
0	5	0
0	3	3
0	1	6

There are seven possible combinations that will result in a total of 15 points.

Section 1: Activity 6

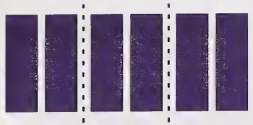
1. To divide 12 by 2 means to find how many groups of 2 there are in 12.

2. a.



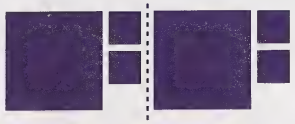
$\therefore\ 8x^2 \div 2 = 4x^2$

b.



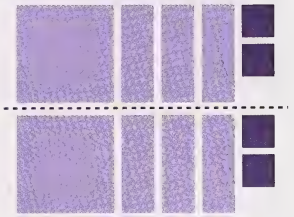
$\therefore\ 6x \div 3 = 2x$

c.



$\therefore\ \left(2x^2 + 4\right) \div 2 = x^2 + 2$

d.

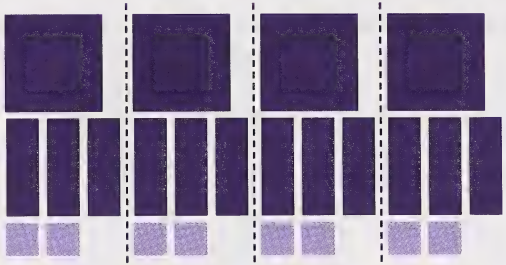


$\therefore\ \left(-2x^2 - 6x + 4\right) \div 2 = -x^2 - 3x + 2$

e.

$\therefore\ (9x + 6) \div 3 = 3x + 2$

f.



$\therefore\ \left(4x^2 + 12x - 8\right) \div 4 = x^2 + 3x - 2$

3. Divide the numerical coefficient of each term by the constant.

4. a. $(6x - 9) \div 3 = \dfrac{6x}{3} - \dfrac{9}{3}$

$= 2x - 3$

b. $\left(4x^2 + 2x - 2\right) \div 2 = \dfrac{4x^2}{2} + \dfrac{2x}{2} - \dfrac{2}{2}$

$= 2x^2 + x - 1$

c. $\dfrac{6x^2 + 9x - 3}{3} = \dfrac{6x^2}{3} + \dfrac{9x}{3} - \dfrac{3}{3}$

$= 2x^2 + 3x - 1$

5. a. $(12y-8)\div 4=\dfrac{12y}{4}-\dfrac{8}{4}$

$=3y-2$

The length of a side is $3y-2$ units.

b. $A=(3y-2)(3y-2)$

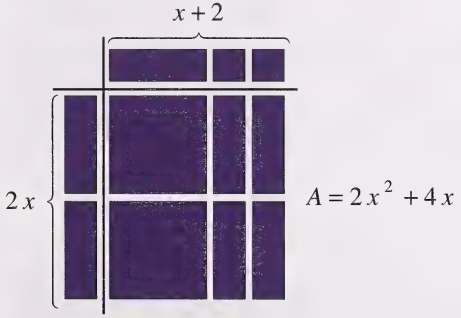
$=(3y)(3y)+(3y)(-2)+(-2)(3y)+(-2)(-2)$

$=9y^2-6y-6y+4$

$=9y^2-12y+4$

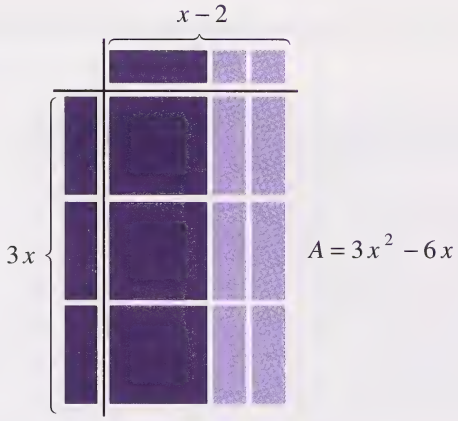
The area of the square is $9y^2-12y+4$ square units.

6. a.



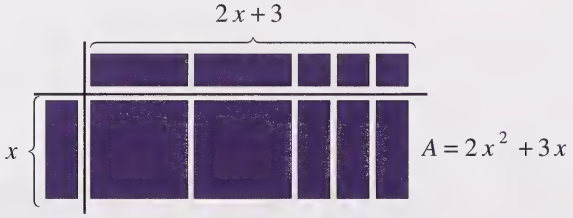
$\therefore\ (2x^2+4x)\div 2x=x+2$

b.



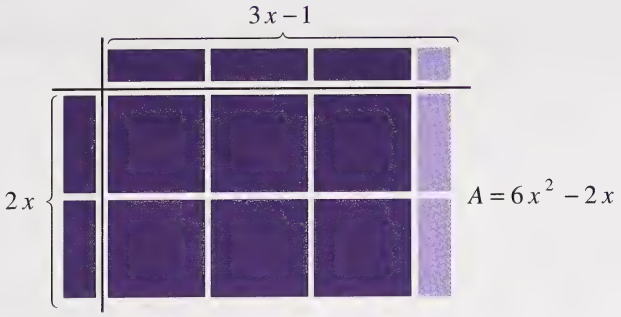
$\therefore\ (3x^2-6x)\div 3x=x-2$

c.



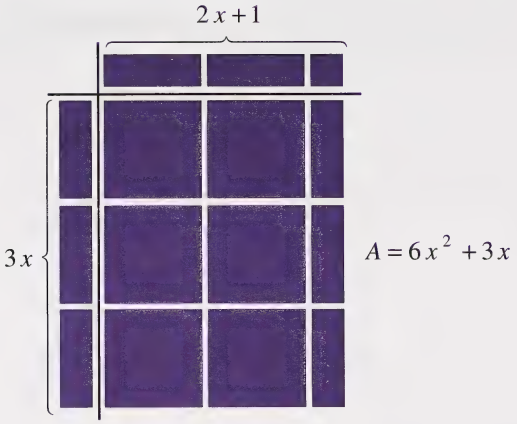
$\therefore\ (2x^2+3x)\div x=2x+3$

d.



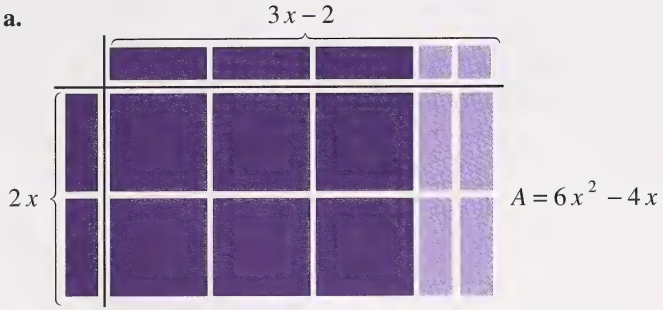
$\therefore \left(6x^2 - 2x\right) \div 2x = 3x - 1$

b.



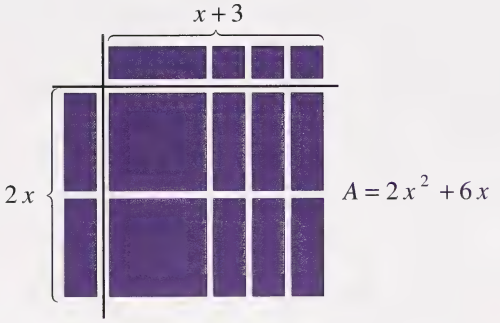
$\therefore \left(6x^2 + 3x\right) \div (3x) = 2x + 1$

7. a.



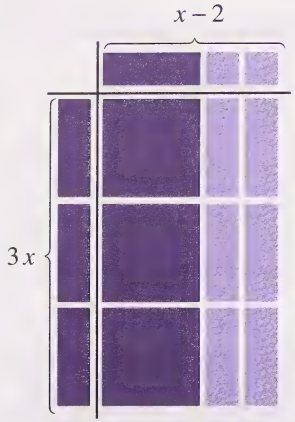
$\therefore \left(6x^2 - 4x\right) \div 2x = 3x - 2$

c.



$\therefore \left(2x^2 + 6x\right) \div 2x = x + 3$

d.



$A = 3x^2 - 6x$

$\therefore \left(3x^2 - 6x\right) \div 3x = x - 2$

8. Divide the numerical coefficient and the literal coefficient (variable) of each term in the polynomial by those of the divisor.

9. a. $\dfrac{6x^2 + 5x}{x} = \dfrac{6x^2}{x} + \dfrac{5x}{x}$

$= 6x + 5$

b. $\dfrac{4x^2 + 6x}{2x} = \dfrac{4x^2}{2x} + \dfrac{6x}{2x}$

$= 2x + 3$

c. $\dfrac{10x^2 + 5x}{5x} = \dfrac{10x^2}{5x} + \dfrac{5x}{5x}$

$= 2x + 1$

10. a. $\dfrac{4x^2 + 2x + 6}{2} = \dfrac{4x^2}{2} + \dfrac{2x}{2} + \dfrac{6}{2}$

$= 2x^2 + x + 3$

Verification

$2\left(2x^2 + x + 3\right) = 2\left(2x^2\right) + 2(x) + 2(3)$

$= 4x^2 + 2x + 6$

b. $\dfrac{3x^2 - 6x}{3x} = \dfrac{3x^2}{3x} + \dfrac{-6x}{3x}$

$= x - 2$

Verification

$3x(x - 2) = 3x(x) + 3x(-2)$

$= 3x^2 - 6x$

c. $\dfrac{5x^3 + 10x^2 - 15x}{5x} = \dfrac{5x^3}{5x} + \dfrac{10x^2}{5x} + \dfrac{-15x}{5x}$

$= x^2 + 2x - 3$

Verification

$5x\left(x^2 + 2x - 3\right) = 5x\left(x^2\right) + 5x(2x) + 5x(-3)$

$= 5x^3 + 10x^2 - 15x$

d. $\dfrac{x^2+4x}{x} = \dfrac{x^2}{x} + \dfrac{4x}{x}$

$= x + 4$

Verification

$x(x+4) = x(x) + x(4)$

$= x^2 + 4x$

11. a. $\dfrac{4x^3+2x^2-6x}{2x} = \dfrac{4x^3}{2x} + \dfrac{2x^2}{2x} + \dfrac{-6x}{2x}$

$= 2x^2 + x - 3$

Verification

LS	RS
$\dfrac{4(1)^3+2(1)^2-6(1)}{2(1)}$	$2(1)^2+1-3$
$= \dfrac{4+2-6}{2}$	$= 2+1-3$
$= \dfrac{0}{2}$	$= 0$
$= 0$	

LS = RS

b. $\dfrac{6x-12}{-3} = \dfrac{6x}{-3} + \dfrac{-12}{-3}$

$= -2x + 4$

Verification

LS	RS
$\dfrac{6(1)-12}{-3}$	$-2(1)+4$
$= \dfrac{6-12}{-3}$	$= -2+4$
$= \dfrac{-6}{-3}$	$= 2$
$= 2$	

LS = RS

c. $\dfrac{4x^2+6x-8}{5} = \dfrac{4x^2}{5} + \dfrac{6x}{5} + \dfrac{-8}{5}$

$= \dfrac{4}{5}x^2 + \dfrac{6}{5}x - \dfrac{8}{5}$

Verification

LS	RS
$\dfrac{4(1)^2+6(1)-8}{5}$	$\dfrac{4}{5}(1)^2 + \dfrac{6}{5}(1) - \dfrac{8}{5}$
$= \dfrac{4+6-8}{5}$	$= \dfrac{4}{5} + \dfrac{6}{5} - \dfrac{8}{5}$
$= \dfrac{2}{5}$	$= \dfrac{4+6-8}{5}$
	$= \dfrac{2}{5}$

LS = RS

d. $\dfrac{8x^3+12x^2-4x}{-4x}=\dfrac{8x^3}{-4x}+\dfrac{12x^2}{-4x}+\dfrac{-4x}{-4x}$

$=-2x^2-3x+1$

Verification

LS	RS
$\dfrac{8(1)^3+12(1)^2-4(1)}{-4(1)}$	$-2(1)^2-3(1)+1$
$=\dfrac{8+12-4}{-4}$	$=-2-3+1$
$=\dfrac{16}{-4}$	$=-4$
$=-4$	

LS = RS

12. a. $\left(6x^3+4x^2+8x\right)\div 3x=\dfrac{6x^3}{3x}+\dfrac{4x^2}{3x}+\dfrac{8x}{3x}$

$=2x^2+\dfrac{4}{3}x+\dfrac{8}{3}$

b. $\left(12x^4+8x^3-16x^2+4x\right)\div(-4x)$

$=\dfrac{12x^4}{-4x}+\dfrac{8x^3}{-4x}+\dfrac{-16x^2}{-4x}+\dfrac{4x}{-4x}$

$=-3x^3-2x^2+4x-1$

c. $\left(x^3-9x\right)\div x=\dfrac{x^3}{x}+\dfrac{-9x}{x}$

$=x^2-9$

d. $\dfrac{-12y^3+9y^2-y}{-3y}=\dfrac{-12y^3}{-3y}+\dfrac{9y^2}{-3y}+\dfrac{-y}{-3y}$

$=4y^2-3y+\dfrac{1}{3}$

e. $\dfrac{24-8m+4m^2}{4}=\dfrac{24}{4}+\dfrac{-8m}{4}+\dfrac{4m^2}{4}$

$=6-2m+m^2$

f. $\dfrac{14a^2b^3+7a^2b^2-7ab^2}{7ab}=\dfrac{14a^2b^3}{7ab}+\dfrac{7a^2b^2}{7ab}+\dfrac{-7ab^2}{7ab}$

$=2ab^2+ab-b$

13. a. $A=\dfrac{3a^2+6a}{3a}$

$=a+2$

b. $A=\dfrac{4c^2-8c}{4c}$

$=c-2$

14. a. $A = \dfrac{6b^2 + 2b}{2b}$

$= 3b + 1$

The length is $3b + 1$.

b. $P = 2(2b) + 2(3b + 1)$

$= 4b + 6b + 2$

$= 10b + 2$

The perimeter is $10b + 2$.

15. a. $\dfrac{6(2)^2(-3)^2}{2(2)(-3)} = \dfrac{6(4)(9)}{2(2)(-3)}$

$= \dfrac{216}{-12}$

$= -18$

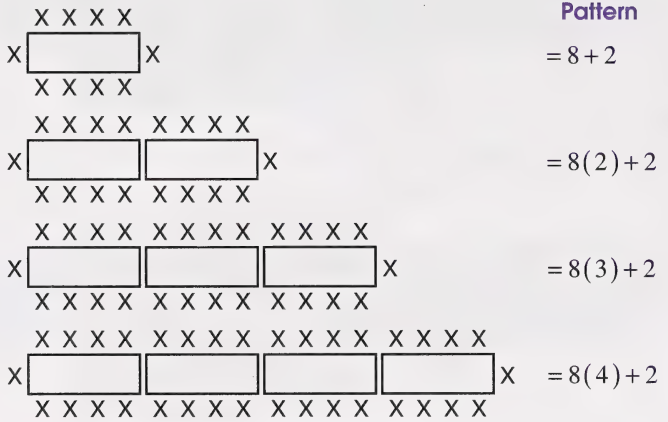
b. $3(2)(-3) = 6(-3)$

$= -18$

16. They are the same because $6x^2y^2 \div 2xy = 3xy$.

Now Try This

17. Find the pattern using one, two, three, and four tables. You may wish to make diagrams.



From the pattern, the number of people that can be seated can be represented by $8n + 2$, where n is the number of tables.

Use the equation to find the number of people that can be seated at 13 tables.

$8n + 2 = 8(13) + 2$

$= 104 + 2$

$= 106$

Therefore, 106 people may be seated at 13 tables.

Section 1: Follow-up Activities

Extra Help

1. a. Represent $3x+6$ in a way that allows you to subtract $2x-2$.

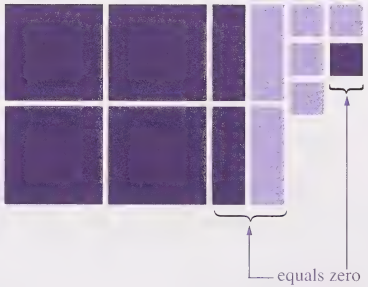
Remove two x-tiles and two -1-tiles.

Show and write an expression for the remaining tiles.

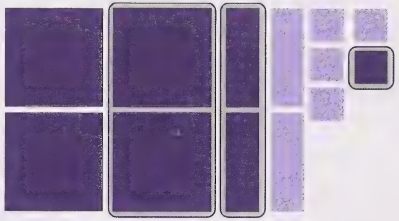
$x+8$

Thus, $(3x+6)-(2x-2)=x+8$.

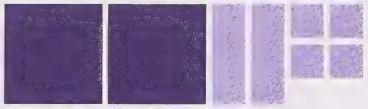
b. Represent $4x^2-3$ in a way that allows you to subtract $2x^2+2x+1$.



Remove two x^2-tiles, two x-tiles, and one 1-tile.

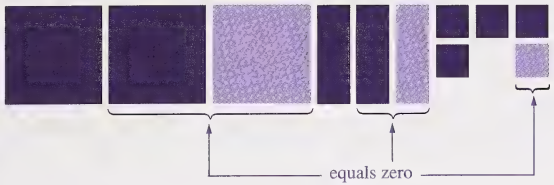


Show and write an expression for the remaining tiles.

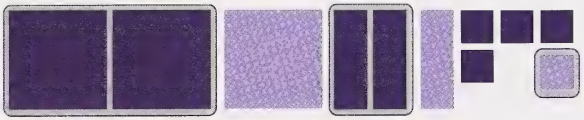


Thus, $\left(4x^2-3\right)-\left(2x^2+2x+1\right)=2x^2-2x-4$.

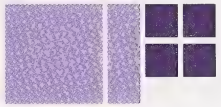
c. Represent $x^2 + x + 3$ in a way that allows you to subtract $2x^2 + 2x - 1$.



Remove two x^2-tiles, two x-tiles, and one -1-tile.



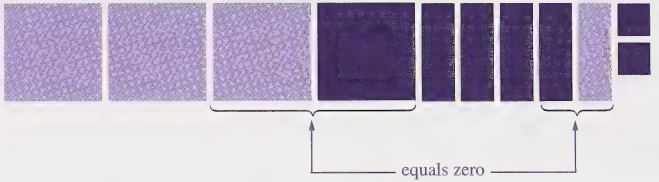
Show and write an expression for the remaining tiles.



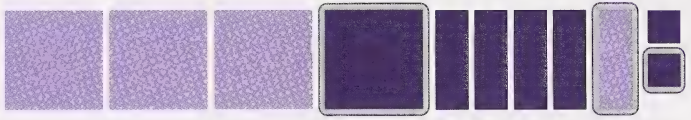
$-x^2 - x + 4$

Thus, $\left(x^2 + x + 3\right) - \left(2x^2 + 2x - 1\right) = -x^2 - x + 4$.

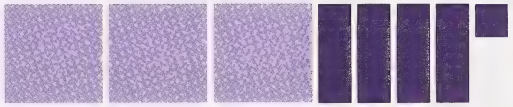
d. Represent $-2x^2 + 3x + 2$ in a way that allows you to subtract $x^2 - x + 1$.



Remove one x^2-tile, one $-x$-tile, and one 1-tile.



Show and write an expression for the remaining tiles.



$-3x^2 + 4x + 1$

Thus, $\left(-2x^2 + 3x + 2\right) - \left(x^2 - x + 1\right) = -3x^2 + 4x + 1$.

2. **a.** x^2+2x-1 **b.** $-2x^2-x+3$

3. **a.** $(3x^2+4x-1)-(x^2-x+2)$

$$=3x^2+4x-1-x^2+x-2$$
$$=3x^2-x^2+4x+x-1-2$$
$$=(3-1)x^2+(4+1)x+(-1-2)$$
$$=2x^2+5x-3$$

b. $(4x+6)-(2x-1)=4x+6-2x+1$

$$=4x-2x+6+1$$
$$=(4-2)x+(6+1)$$
$$=2x+7$$

c. $(-2x^2+3x+4)-(5x-1)=-2x^2+3x+4-5x+1$

$$=-2x^2+3x-5x+4+1$$
$$=-2x^2+(3-5)x+(4+1)$$
$$=-2x^2-2x+5$$

d. $(3x^2+5x)-(x^2+1)=3x^2+5x-x^2-1$

$$=3x^2-x^2+5x-1$$
$$=(3-1)x^2+5x-1$$
$$=2x^2+5x-1$$

4. The answer to the puzzle is **HE'S REALLY A FUNGI (He's really a fun guy)**.

Enrichment

1. The algebra tiles given in the Appendix are x^2-tiles, x-tiles, and 1-tiles. Using these tiles, you can model polynomials of degree 0, 1, or 2. A polynomial of degree 3 would require more than a square tile. A cube could be used to model a product of degree 3. However, this is difficult to produce and show using a two-dimensional page.

2. **a.** $(x+4)(x-3)=(x)(x)+(x)(-3)+(4)(x)+(4)(-3)$

$$=x^2-3x+4x-12$$
$$=x^2+x-12$$

b. $(x-7)(x+2)=(x)(x)+(x)(2)+(-7)(x)+(-7)(2)$

$$=x^2+2x-7x-14$$
$$=x^2-5x-14$$

c. $(2x+5)(x+2)=(2x)(x)+(2x)(2)+(5)(x)+(5)(2)$

$$=2x^2+4x+5x+10$$
$$=2x^2+9x+10$$

d. $(3x-1)(2x+5)$

$=(3x)(2x)+(3x)(5)+(-1)(2x)+(-1)(5)$

$=6x^2+15x-2x-5$

$=6x^2+13x-5$

e. $(x-8)(x+8)=(x)(x)+(x)(8)+(-8)(x)+(-8)(8)$

$=x^2+8x-8x-64$

$=x^2-64$

f. $(2x+1)(2x+1)$

$=(2x)(2x)+(2x)(1)+(1)(2x)+(1)(1)$

$=4x^2+2x+2x+1$

$=4x^2+4x+1$

g. $(3x+4)(2x-3)$

$=(3x)(2x)+(3x)(-3)+(4)(2x)+(4)(-3)$

$=6x^2-9x+8x-12$

$=6x^2-x-12$

h. $(x+8)(x-3)=(x)(x)+(x)(-3)+(8)(x)+(8)(-3)$

$=x^2-3x+8x-24$

$=x^2+5x-24$

i. $(2x-5)(x+4)$

$=(2x)(x)+(2x)(4)+(-5)(x)+(-5)(4)$

$=2x^2+8x-5x-20$

$=2x^2+3x-20$

j. $(x-10)(x+6)$

$=(x)(x)+(x)(6)+(-10)(x)+(-10)(6)$

$=x^2+6x-10x-60$

$=x^2-4x-60$

k. $(x-9)(x+9)=(x)(x)+(x)(9)+(-9)(x)+(-9)(9)$

$=x^2+9x-9x-81$

$=x^2-81$

l. $(x+7)^2=(x+7)(x+7)$

$=(x)(x)+(x)(7)+(7)(x)+(7)(7)$

$=x^2+7x+7x+49$

$=x^2+14x+49$

3. a. $(2y+5)^2=(2y+5)(2y+5)$

$=(2y)(2y)+(2y)(5)+(5)(2y)+(5)(5)$

$=4y^2+10y+10y+25$

$=4y^2+20y+25$

b. $(y+3)^2-(y+2)(y-4)$

$=(y+3)(y+3)-(y+2)(y-4)$

$=[(y)(y)+(y)(3)+(3)(y)+(3)(3)]$

$\quad -[(y)(y)+(y)(-4)+(2)(y)+(2)(-4)]$

$=\left(y^2+3y+3y+9\right)-\left(y^2-4y+2y-8\right)$

$=\left(y^2+6y+9\right)-\left(y^2-2y-8\right)$

$=y^2+6y+9-y^2+2y+8$

$=8y+17$

c. $3(y+5)(y-2)=3\left(y^2-2y+5y-10\right)$

$=3\left(y^2+3y-10\right)$

$=3y^2+9y-30$

d. $(x+4)^2-(x-4)^2$

$=(x+4)(x+4)-(x-4)(x-4)$

$=\left(x^2+4x+4x+16\right)-\left(x^2-4x-4x+16\right)$

$=\left(x^2+8x+16\right)-\left(x^2-8x+16\right)$

$=x^2+8x+16-x^2+8x-16$

$=16x$

e. $(x+3)(x+5)+2(x+3)=\left(x^2+5x+3x+15\right)+(2x+6)$

$=x^2+8x+15+2x+6$

$=x^2+10x+21$

f. $(x-3)^2+(x+2)(x-2)$

$=(x-3)(x-3)+(x+2)(x-2)$

$=\left(x^2-3x-3x+9\right)+\left(x^2-2x+2x-4\right)$

$=x^2-6x+9+x^2-4$

$=2x^2-6x+5$

Section 2: Activity 1

1. a. You should have found four different rectangles:

- 1 by 24
- 2 by 12
- 3 by 8
- 4 by 6

Note: 1 by 24 and 24 by 1, 3 by 8 and 8 by 3, and so on are the same.

b. They are the factors of 24.

2.

Length	Width	Area
3 m	2 m	$6\,m^2$
6 m	1 m	
10 m	2 m	$20\,m^2$
5 m	4 m	
20 m	1 m	
16 m	1 m	$16\,m^2$
4 m	4 m	
8 m	2 m	
5 m	1 m	$5\,m^2$

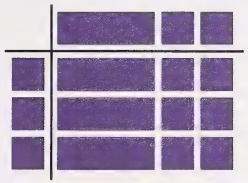
3. **a.** The factors of 6 are 1, 2, 3, and 6.

b. The factors of 20 are 1, 2, 4, 5, 10, and 20.

c. The factors of 16 are 1, 2, 4, 8, and 16.

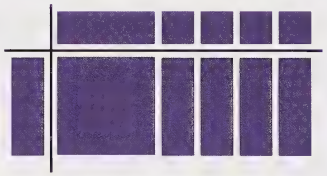
d. The factors of 5 are 1 and 5.

4. **a.**



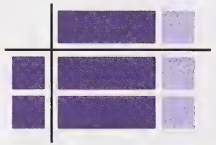
The product is $3x+6$.

b.



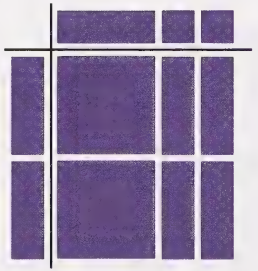
The product is x^2+4x.

c.



The product is $2x-2$.

d.



The product is $2x^2+4x$.

5. **a.** 3 and $x+2$ **b.** x and $x+4$

c. 2 and $x-1$ **d.** $2x$ and $x+2$

6. You can find the other factor by dividing the polynomial by the known factor.

7. **a.** $4x+6=2(\ 2x+3\ )$ **b.** $x^2+3x=x(\ x+3\ )$

c. $5x-10=5(\ x-2\ )$ **d.** $2x^2+6x=2x(\ x+3\ )$

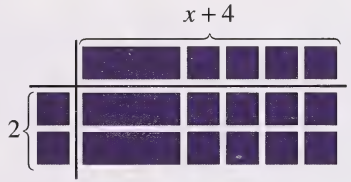
e. $2x^2+4x+10=2(\ x^2+2x+5\ )$

f. $4x^2+8x+12=4(\ x^2+2x+3\ )$

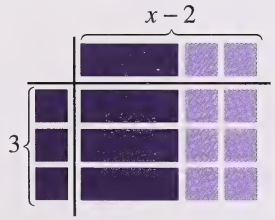
Section 2: Activity 2

1. The following shows how algebra tiles are used to find each dimension.

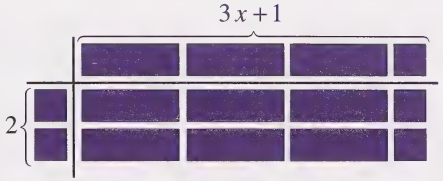
- $2x+8$



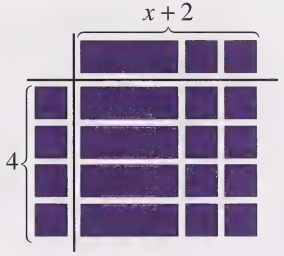
- $3x-6$



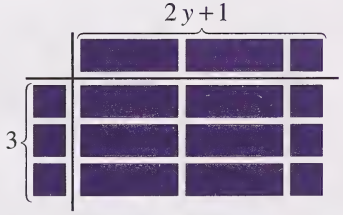
- $6x+2$



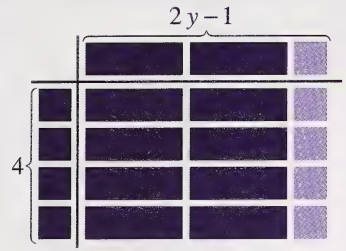
- $4x+8$



- $6y+3$



- $8y-4$

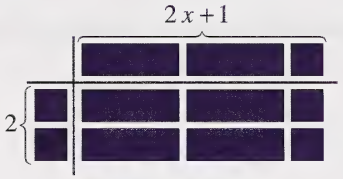


The completed chart is as follows:

Area	Length	Width	Area as a Product
$2x+8$	$x+4$	2	$2(x+4)$
$3x-6$	$x-2$	3	$3(x-2)$
$6x+2$	$3x+1$	2	$2(3x+1)$
$4x+8$	$x+2$	4	$4(x+2)$
$6y+3$	$2y+1$	3	$3(2y+1)$
$8y-4$	$2y-1$	4	$4(2y-1)$

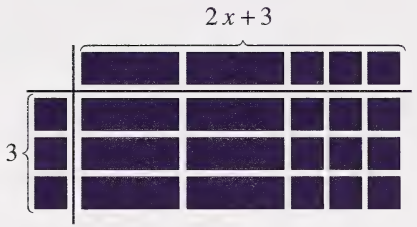
2. The width represents the common factor in each case.

3. a.



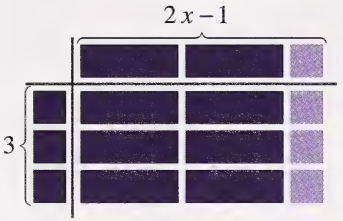
$\therefore\ 4x+2=2(2x+1)$

b.



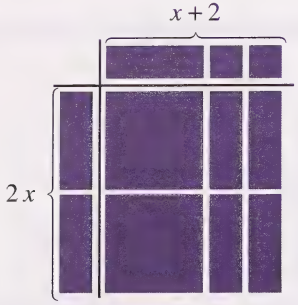
$\therefore\ 6x+9=3(2x+3)$

c.



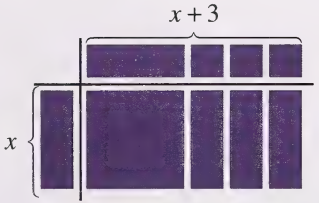
$\therefore\ 6x-3=3(2x-1)$

d.



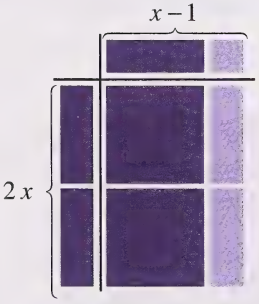
$\therefore\ 2x^2 + 4x = 2x(x+2)$

e.



$\therefore\ x^2 + 3x = x(x+3)$

f.



$\therefore\ 2x^2 - 2x = 2x(x-1)$

4. **a.** 2 **b.** 3 **c.** 3

d. $2x$ **e.** x **f.** $2x$

5. You find the other factor by dividing the greatest common factor into each term of the polynomial.

6. **a.**

$15 = 3 \times 5$

$20 = 2 \times 2 \times 5$

$35 = 5 \times 7$

GCF = 5

b.

$12 = 2 \times 2 \times 3$

$-16 = -2 \times 2 \times 2 \times 2$

$20 = 2 \times 2 \times 5$

GCF = 2(2)
= 4

c.

$6x = 2 \times 3 \times x$

$8x^2 = 2 \times 2 \times 2 \times x \times x$

$4x^3 = 2 \times 2 \times x \times x \times x$

GCF = 2(x)
= 2x

d.

$-9x = -3 \times 3 \times x$

$15x^2 = 5 \times 3 \times x \times x$

$-12x^3 = 2 \times -2 \times 3 \times x \times x \times x$

GCF = 3(x)
= 3x

e. $24z^4 = 2 \times 2 \times 2 \times 3 \times z \times z \times z \times z$

$30z^3 = \quad 2 \times 3 \times 5 \times z \times z \times z$

$-12z^2 = -2 \times 2 \times 3 \quad \times z \times z$

$\text{GCF} = (2)(3)(z)(z)$

$= 6z^2$

f. $9ab^2c = \quad 3 \times 3 \times a \times b \times b \times c$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

$21b^2c = \quad 3 \times 7 \times b \times b \times c$

$\text{GCF} = 3(b)$

$= 3b$

7. a. $4a^2 + 6a = 2a(2a + 3)$

b. $7a^3 - 14a^2 + 28a = 7a(a^2 - 2a + 4)$

c. $18b + 3b^2 = 3b(6 + b)$

d. $12b^2 + 20b^3 - 24b^4 = 4b^2(3 + 5b - 6b^2)$

e. $10b^3 + 8b^2 - 12b + 6 = 2(5b^3 + 4b^2 - 6b + 3)$

8. a. $2x(x + 4)$

Verification

$2x(x+4) = (2x)(x) + (2x)(4)$

$= 2x^2 + 8x$

b. $ab(4b - 7a)$

Verification

$ab(4b - 7a) = (ab)(4b) + (ab)(-7a)$

$= 4ab^2 - 7a^2b$

c. $2(2a^2 + 3a + 4)$

Verification

$2(2a^2 + 3a + 4) = (2)(2a^2) + (2)(3a) + (2)(4)$

$= 4a^2 + 6a + 8$

d. $7y^2(1 + 2y)$

Verification

$7y^2(1+2y) = (7y^2)(1) + (7y^2)(2y)$

$= 7y^2 + 14y^3$

e. $4(a-4)$

Verification

$$\begin{aligned}4(a-4) &= (4)(a)+(4)(-4)\\ &= 4a-16\end{aligned}$$

f. $2xy\left(1+3xy-4x^2y^2\right)$

Verification

$$\begin{aligned}&2xy\left(1+3xy-4x^2y^2\right)\\ &\quad=(2xy)(1)+(2xy)(3xy)+(2xy)\left(-4x^2y^2\right)\\ &\quad=2xy+6x^2y^2-8x^3y^3\end{aligned}$$

g. $a(b+c)$

Verification

$$\begin{aligned}a(b+c) &= (a)(b)+(a)(c)\\ &= ab+ac\end{aligned}$$

h. $a(b+c-2d+4e)$

Verification

$$\begin{aligned}&a(b+c-2d+4e)\\ &\quad=(a)(b)+(a)(c)+(a)(-2d)+(a)(4e)\\ &\quad=ab+ac-2ad+4ae\end{aligned}$$

9. $8x^2+6x=2x(4x+3)$

$2x^2+10x=2x(x+5)$

common factor of the polynomials

Length of the common side is $2x$.

10. $P=2(\ell+w)$

11. $3x^2+x=x(3x+1)$

The length of the pen is $3x+1$ and the width is x.

12. **a.** $m+n$ **b.** $a-3$ **c.** $2x+3$

d. $d-1$ **e.** $a+3$ **f.** $x-1$

13. **a.** $(x-7)(x+3)$ **b.** $(y+3)(2y-1)$

c. $(a+7)(a+3)$ **d.** $(2n+3)(n+3)$

e. $(m+1)(m-3)$ **f.** $(t+3)(2t+5)$

14. **a.** $(a+3)(b-c)$ **b.** $(y+4)(2y+3)$

c. $(b+1)(2a+c+2)$ **d.** $(z-y)(6+x)$

e. $y(x+2)+2(x+2)=(x+2)(y+2)$

f. $a(a+8)-3(a+8)=(a+8)(a-3)$

g. $\left(x^2+5x\right)+(6x+30)=x(x+5)+6(x+5)$
$=(x+5)(x+6)$

h. $\left(y^2+4y\right)+(3y+12)=y(y+4)+3(y+4)$
$=(y+4)(y+3)$

Now Try This

15. **a.** 16 **b.** $8^2=64$

c. $20^2=400$ **d.** $2n-1$

e. The sum is equal to n^2.

16. **a.** $5!=5\times4\times3\times2\times1$
$=120$

b. $7\times6!=7!$

c. $5!\div4!=\dfrac{5!}{4!}$
$=\dfrac{5\times\cancel{4!}}{\cancel{4!}}$
$=5$

d. $\dfrac{10!\times8!}{9!\times7!}=\dfrac{10!}{9!}\times\dfrac{8!}{7!}$
$=\dfrac{10\times\cancel{9!}}{\cancel{9!}}\times\dfrac{8\times\cancel{7!}}{\cancel{7!}}$
$=10\times8$
$=80$

e. i. $(7-1)!=6!$
$=720$

ii. $\left(\sqrt{100}\right)!=(10)!$
$=3\,628\,800$

iii. $8!-6!=40\,320-720$
$=39\,600$

Section 2: Activity 3

1. The length is $x+3$, and the width is $x+2$.

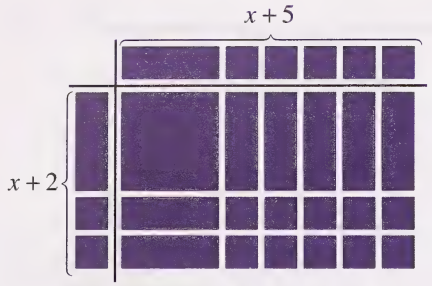
2. The factors of the trinomial x^2+5x+6 are $x+3$ and $x+2$.

3. The length and width of the rectangle are $x-2$ and $x-1$ respectively. The factors of the trinomial are $x-2$ and $x-1$.

4. The length and width of the rectangle are $x-4$ and $x+1$ respectively. The factors of the trinomial are $x-4$ and $x+1$.

5. **a.** $x^2+7x+10$

b.



The length and width of the rectangular model are $x+5$ and $x+2$ respectively.

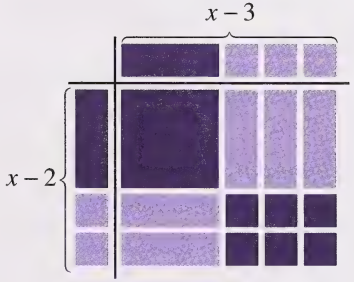
Therefore, $x^2+7x+10=(x+5)(x+2)$.

c. $(x+5)(x+2)=x^2+2x+5x+10$

$=x^2+7x+10$

6. a. x^2-5x+6

b.



The length and width of the rectangular model are $x-3$ and $x-2$ respectively.

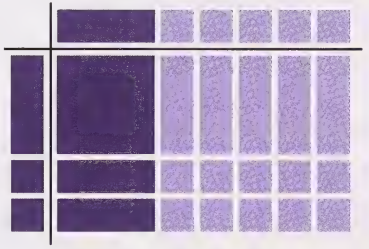
Therefore, $x^2-5x+6=(x-3)(x-2)$.

c. $(x-3)(x-2)=x^2-2x-3x+6$

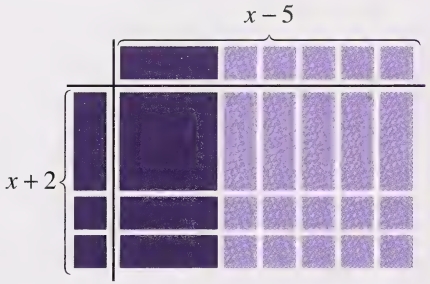
$=x^2-5x+6$

7. a. $x^2-3x-10$

b. The completed rectangle is as follows:



Draw in the length and width as follows:



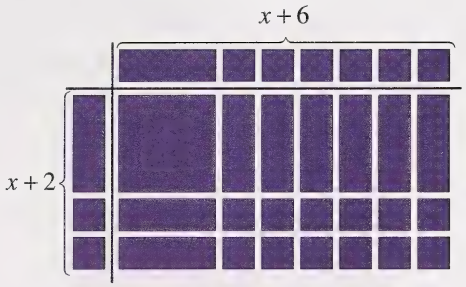
The length and width of the rectangular model are $x-5$ and $x+2$ respectively.

Therefore, $x^2-3x-10=(x-5)(x+2)$.

c. $(x-5)(x+2)=x^2+2x-5x-10$

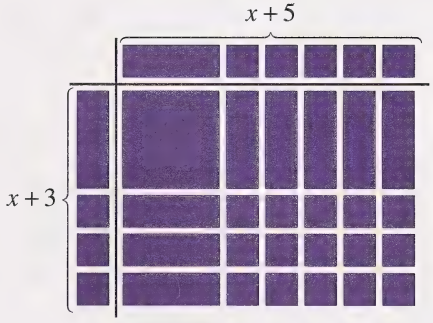
$=x^2-3x-10$

8. a.



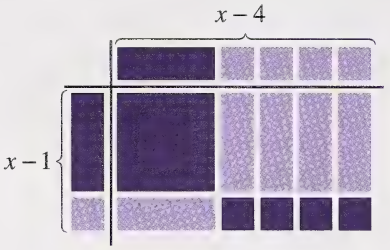
$\therefore\ x^2+8x+12=(x+6)(x+2)$

b.



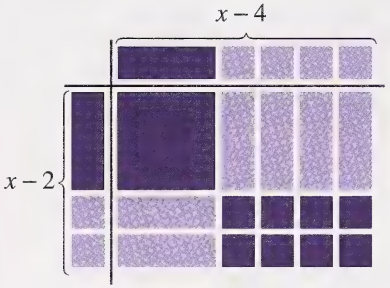
$\therefore\ x^2+8x+15=(x+5)(x+3)$

c.



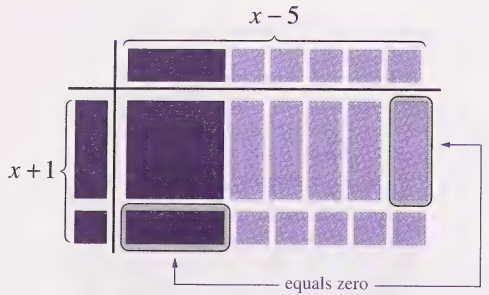
$\therefore\ x^2-5x+4=(x-4)(x-1)$

d.



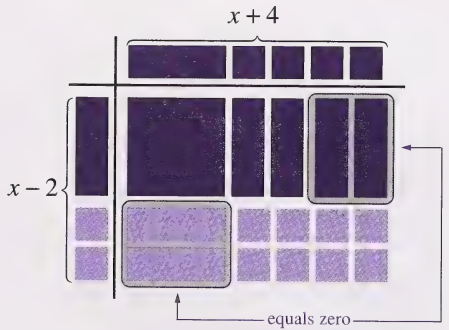
$\therefore\ x^2-6x+8=(x-4)(x-2)$

e.



$\therefore\ x^2 - 4x - 5 = (x-5)(x+1)$

f.



$\therefore\ x^2 + 2x - 8 = (x+4)(x-2)$

9. Annette's arrangement is correct because her tiles form a rectangle. When using algebra tiles for factoring, a rectangle or square must be formed completely.

10. When the second term of each factor are added together, you get the coefficient of the middle term of the trinomial. When the second term of each factor are multiplied, you get the last term of the trinomial.

11. The factors of 16 are 16 and 1, 8 and 2, and 4 and 4.

12. The factors that add up to 10 are 8 and 2.

13. a. $x^2 + 11x + 30 = (x + 5)(x + 6)$

b. $x^2 - 9x + 20 = (x - 5)(x - 4)$

c. $n^2 - 3n - 18 = (n - 6)(n + 3)$

d. $y^2 + 5y - 14 = (y + 7)(y - 2)$

14. a. $x^2 + 13x + 36 = (x+4)(x+9)$

b. $x^2 - 14x - 32 = (x-16)(x+2)$

c. $y^2 - 11y + 24 = (y-3)(y-8)$

15. a. $(x-10)(x-2)$

Verification

$$(x-10)(x-2) = x^2 - 2x - 10x + 20$$
$$= x^2 - 12x + 20$$

b. $(n-6)(n+2)$

Verification

$$(n-6)(n+2) = n^2 + 2n - 6n - 12$$
$$= n^2 - 4n - 12$$

c. $(y+12)(y-1)$

Verification

$$(y+12)(y-1) = y^2 - y + 12y - 12$$
$$= y^2 + 11y - 12$$

d. $(m-3)(m-3)$

Verification

$$(m-3)(m-3) = m^2 - 3m - 3m + 9$$
$$= m^2 - 6m + 9$$

e. $(p+4)(p+4)$

Verification

$$(p+4)(p+4) = p^2 + 4p + 4p + 16$$
$$= p^2 + 8p + 16$$

f. $(b+8)(b-7)$

Verification

$$(b+8)(b-7) = b^2 - 7b + 8b - 56$$
$$= b^2 + b - 56$$

16. a. $3x^2 + 15x + 12 = 3\left(x^2 + 5x + 4\right)$
$$= 3(x+4)(x+1)$$

b. $2x^2 - 20x + 50 = 2\left(x^2 - 10x + 25\right)$
$$= 2(x-5)(x-5)$$

c. $2x^3 + 14x^2 + 12x = 2x\left(x^2 + 7x + 6\right)$
$$= 2x(x+6)(x+1)$$

d. $3x^3 + 21x^2 - 54x = 3x\left(x^2 + 7x - 18\right)$
$$= 3x(x+9)(x-2)$$

e. $6y^2 + 12y + 6 = 6\left(y^2 + 2y + 1\right)$
$$= 6(y+1)(y+1)$$

f. $x^2y + 3xy - 40y = y\left(x^2 + 3x - 40\right)$
$$= y(x+8)(x-5)$$

g. $4x^2 - 28x + 48 = 4\left(x^2 - 7x + 12\right)$

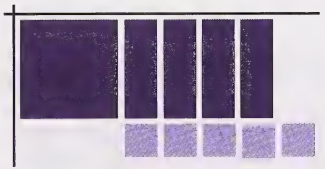
$= 4(x-3)(x-4)$

h. $x^3 + 2x^2 + x = x\left(x^2 + 2x + 1\right)$

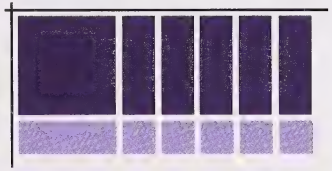
$= x(x+1)(x+1)$

Now Try This

17. a. Draw the tiles for $x^2 + 4x - 5$.



Add one x-tile and one $-x$-tile.



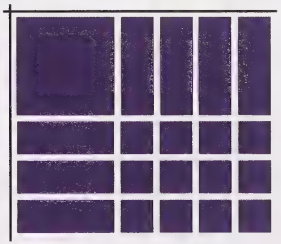
b. The dimensions are $x+5$ and $x-1$.

c. Complete the rectangle by adding as many pairs of positive and negative x-tiles or 1-tiles as are necessary.

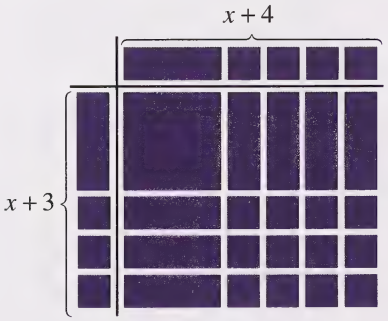
Section 2: Follow-up Activities

Extra Help

1. a. To factor $x^2 + 7x + 12$, start with four and three x-tiles because 4 and 3 are factors of 12 that add up to 7.

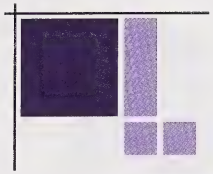


Draw in the factors.

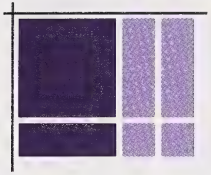


$\therefore\ x^2 + 7x + 12 = (x+3)(x+4)$

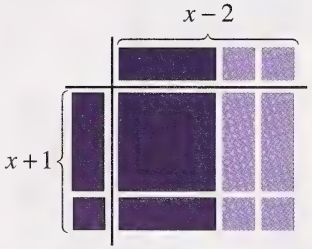
b. To factor $x^2 - x - 2$, start with this.



Add one x-tile and one $-x$-tile to complete the rectangle.



Draw in the factors.

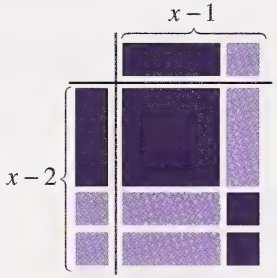


$\therefore\ x^2 - x - 2 = (x+1)(x-2)$

Remember: Two plus one equals one , or $-2x + x = -x$.

c. To factor $x^2 - 3x + 2$, start with this.

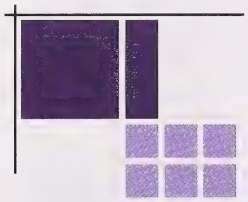
Draw in the factors.



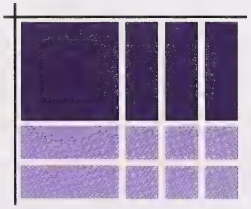
$\therefore\ x^2 - 3x + 2 = (x-2)(x-1)$

Recall: A tile like is positive. A tile like is negative. Therefore, × = , since $(-)\times(-)=(+)$. The same applies to the other shapes.

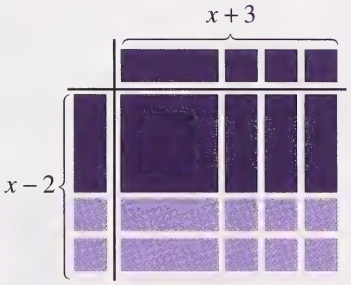
d. To factor x^2+x-6, start with this.



Add two x-tiles and two $-x$-tiles to complete the rectangle.



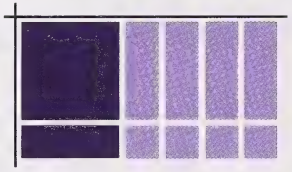
Draw in the factors.



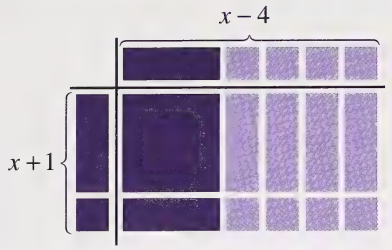
$\therefore\ x^2+x-6=(x-2)(x+3)$

e. To factor x^2-3x-4, start with this.

Add one x-tile and one $-x$-tile to complete the rectangle.

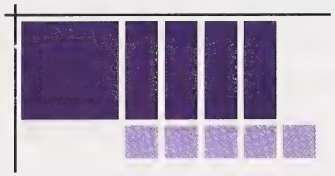


Draw in the factors.

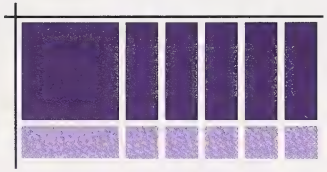


$\therefore\ x^2 - 3x - 4 = (x+1)(x-4)$

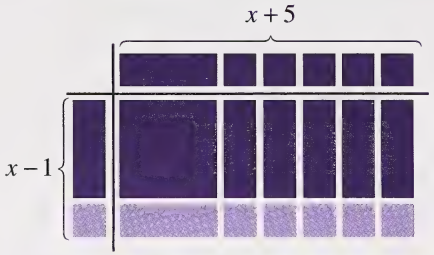
f. To factor $x^2 + 4x - 5$, start with this.



Add one x-tile and one $-x$-tile to complete the rectangle.



Draw in the factors.



$\therefore\ x^2 + 4x - 5 = (x-1)(x+5)$

2. The answer to the puzzle is **FROM A FROG PAWNED** (pond).

Enrichment

1. a. $a^2 - 6a + 2a - 12 = \left(a^2 - 6a\right) + (2a - 12)$

$= a(a-6) + 2(a-6)$

$= (a-6)(a+2)$

b. $ab^2 + abc + db + dc = \left(ab^2 + abc\right) + (db + dc)$

$= ab(b+c) + d(b+c)$

$= (b+c)(ab+d)$

c. $xy+3x-y-3=(xy+3x)+(-y-3)$
$=x(y+3)-1(y+3)$
$=(y+3)(x-1)$

d. $x^2+15+3x+5x=x^2+3x+5x+15$
$=\left(x^2+3x\right)+(5x+15)$
$=x(x+3)+5(x+3)$
$=(x+3)(x+5)$

e. $ma-2n+na-2m=ma-2m+na-2n$
$=(ma-2m)+(na-2n)$
$=m(a-2)+n(a-2)$
$=(a-2)(m+n)$

f. $m^2-2m-5m+10=\left(m^2-2m\right)+(-5m+10)$
$=m(m-2)-5(m-2)$
$=(m-2)(m-5)$

g. $abc+adc+4be+4de=(abc+adc)+(4be+4de)$
$=ac(b+d)+4e(b+d)$
$=(b+d)(ac+4e)$

h. $ad+bd+2ac+2bc=(ad+bd)+(2ac+2bc)$
$=d(a+b)+2c(a+b)$
$=(a+b)(d+2c)$

i. $n^2+11n+24=n^2+3n+8n+24$
$=n(n+3)+8(n+3)$
$=(n+3)(n+8)$

2. a. $y^2+9y+14=y^2+2y+7y+14$
$=\left(y^2+2y\right)+(7y+14)$
$=y(y+2)+7(y+2)$
$=(y+2)(y+7)$

b. $x^2-7x+12=x^2-3x-4x+12$
$=\left(x^2-3x\right)+(-4x+12)$
$=x(x-3)-4(x-3)$
$=(x-3)(x-4)$

c. $a^2+13a+36=a^2+9a+4a+36$
$=\left(a^2+9a\right)+(4a+36)$
$=a(a+9)+4(a+9)$
$=(a+9)(a+4)$

d. $n^2+9n+20=n^2+5n+4n+20$

$=\left(n^2+5n\right)+(4n+20)$

$=n(n+5)+4(n+5)$

$=(n+5)(n+4)$

e. $b^2-10b+21=b^2-7b-3b+21$

$=\left(b^2-7b\right)+(-3b+21)$

$=b(b-7)-3(b-7)$

$=(b-7)(b-3)$

f. $c^2-2c-15=c^2-5c+3c-15$

$=\left(c^2-5c\right)+(3c-15)$

$=c(c-5)+3(c-5)$

$=(c-5)(c+3)$

g. $m^2+12m+27=m^2+9m+3m+27$

$=\left(m^2+9m\right)+(3m+27)$

$=m(m+9)+3(m+9)$

$=(m+9)(m+3)$

h. $a^2-13a+30=a^2-10a-3a+30$

$=\left(a^2-10a\right)+(-3a+30)$

$=a(a-10)-3(a-10)$

$=(a-10)(a-3)$

i. $y^2+9y+8=y^2+8y+y+8$

$=\left(y^2+8y\right)+(y+8)$

$=y(y+8)+1(y+8)$

$=(y+8)(y+1)$

j. $x^2+9x-22=x^2+11x-2x-22$

$=\left(x^2+11x\right)+(-2x-22)$

$=x(x+11)-2(x+11)$

$=(x+11)(x-2)$

Puzzles

How Does a Hawaiian Baritone Laugh?

Simplify each expression below. Find your answer at the bottom of the page and cross out the letter above it. When you finish, the answer to the title question will remain.

(1) $-3x+9x$

(2) $2y-10y$

(3) $-6x+x$

(4) $12y-y$

(5) $-4x-5x$

(6) $8y-(-8y)$

(7) $-x-(-10x)$

(8) $-2y+7y+4$

(9) $5x+7+x-9x$

(10) $-8y-2y-4+4y$

(11) $6x-(-3x)+x-6$

(12) $4x+2y+4x-5y$

(13) $6x+8y-3-7y$

(14) $-6x-2y+8+5x-1$

(15) $9-3x-(-8y)+9x-y$

(16) $x-4y-12-5y+8y$

(17) $3x+7-7y+2x-3y-1$

(18) $-9x-y+1+5y+5x-10$

(19) $-x+8+6x-4y-8x+3$

(20) $4x-7+y-7x-(-3y)$

(21) $8x-5y-x+9-y$

H	A	B	S	U	L	E	A	T	N	R	E	O	L	C	M	F	E	W	H	D	I	S	T	A	N	G
$7x-6y+9$	$7x-8y+7$	$8x-3y$	$-3x+4y-7$	$5y+4$	$-4x+7y-6$	$6x$	$10x-6$	$6x+7y+9$	$-9x$	$-3x+7$	$-3x-4y+11$	$-3x+5y-1$	$-8y$	$-x-2y+7$	$16y$	$11y$	$-6y-4$	$x-3y-10$	$6x-6$	$-5x$	$9x$	$6x+y-3$	$5x-10y+6$	$5x-5y+4$	$x-y-12$	$-4x+4y-9$

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What Do You Call Two Railroad Trains After a Head-on Collision?

First, **simplify** each expression below. Then **evaluate** the expression if $a=3$, $b=-2$, $c=-6$. Find the simplified expression in the answer column and notice the letter next to it. Find the value of the expression at the bottom of the page and write this letter above it.

(1) $9a+3-2a$

(2) $8-5b-1$

(3) $-4b-6+20b-3$

(4) $2-(-8c)+24-7c$

(5) $5a-9b+a-6b$

(6) $3b+11c-4b-c$

(7) $9a-1+8c-8a+c$

(8) $12c+5a+7+(-13c)+4a$

(9) $-15-6c+3b-6c+9-2b$

(10) $3a+7b+2c-a-4b$

(11) $-8a-b-(-6c)-2a-b-5c$

(12) $b-4c+3a-c-9b-4a$

(13) $-3c+7a+5+17b+2c+b+(-7a)$

(14) $2-a-(-b)+c+(-a)-b-(-c)$

(B) $16b-3c+5$

(L) $c+26$

(E) $7a+3$

(N) $18b-c+5$

(T) $b-12c-6$

(A) $-5b+7$

(A) $-b+10c$

(W) $16b-9$

(E) $9a-c+7$

(K) $-a-8b-5c$

(R) $-2a+2c+2$

(E) $6a-15b$

(S) $b-10c+6$

(C) $2a+3b+2c$

(D) $a+9c-1$

(H) $-a-10b-8c$

(G) $-10a-2b+c$

−58	−41	−16	24	−12	43	48	−52	64	17	−25	−32	20	40

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Daffynition Decoder

For each exercise below, subtract the second polynomial from the first. Find your answer in the answer column and notice the letter next to it. Each time the exercise number appears in the code, write this letter above it. Keep working and you will decode the "de-fun-itions."

(1) $(7x+4)-(2x+9)$

(2) $(3x+12)-(5x-6)$

(3) $(-4x^2+10)-(6x^2-9)$

(4) $(2x^2+3x+8)-(x^2+5x-1)$

(5) $(-x^2+9x-2)-(9x^2-4x+4)$

(6) $(3x^2+7x+1)-(8+5x+x^2)$

(7) $(4x^3+6x^2-8x)-(x^3-2x^2+12x)$

(8) $(x^3+2x^2+5x)-(3x^2-x-7)$

(9) $(x^4+8x^2-1)-(x^2-3x^3+x^4)$

(10) $(5x^4-2x^2)-(3x-2x^2-4x^3+6x^4)$

(11) $(3x^2+7xy-2y^2)-(x^2-6xy+2y^2)$

(12) $(-x^2-9xy+5y^2)-(4x^2-2xy-y^2)$

(13) $(4x^2y-3xy^2)-(3x^2y-8xy^2)$

1. Romantic: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

11 13 8 12 11 1 8 11 13 8 13 10 3 5 12

2. American: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

11 2 11 9 9 6 5 7 13 12 11 8 13 3 4

Answers:

(M) $-x^4+4x^3-7x^2$

(S) $-x^4+4x^3-3x$

(U) $3x^3+5x^2+7$

(L) $5x-5$

(E) $-10x^2+19$

(F) $2x^2+2x-19$

(C) $-10x^2+13x-6$

(H) $-2x+18$

(T) $-5x^2-7xy+6y^2$

(O) $3x^3+8x^2-20x$

(P) $3x^3+7x^2-1$

(R) x^2-2x+9

(A) $2x^2+13xy-4y^2$

(N) x^2y+5xy^2

(Y) $2x^2+2x-7$

(B) $-5x^2-6xy+7y^2$

(I) x^3-x^2+6x+7

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Why Is a Stick of Gum Like a Sneeze?

For each exercise, multiply the two polynomials. Find your answer in the set of answers under the exercise. Cross out the letter above your answer. When you finish, the answer to the title question will remain!

(1) $(x+3)(x+5)$							
(2) $(x+2)(x+9)$							
(3) $(x-8)(x+1)$							
(4) $(x-3)(x-6)$							
(5) $(2x+9)(x-2)$							
(6) $(3x+1)(2x+4)$							
B	E	S	I	A	U	T	N
x^2-7x-8	$x^2+8x+15$	$6x^2+14x+4$	$6x^2+7x+4$	$x^2-9x+18$	$x^2+11x+18$	$x^2-13x+18$	$2x^2+5x-18$

(7) $(4a-7)(3a-2)$							
(8) $(2a+5)(2a-5)$							
(9) $(6a-1)(2a+4)$							
(10) $(a+2b)(4a+b)$							
(11) $(5a+3b)(a-4b)$							
(12) $(3a-8b)(2a-b)$							
T	I	S	E	R	A	N	O
$4a^2+9ab+2b^2$	$6a^2-19ab+8b^2$	$5a^2-11ab-12b^2$	$12a^2+22a-4$	$4a^2-25$	$4a^2+4ab+3b^2$	$5a^2-17ab-12b^2$	$12a^2-29a+14$

(13) $(n+2)(n^2+5n-3)$									
(14) $(3n-1)(2n^2+4n+4)$									
(15) $(2n+3)(6n^2-2n+1)$									
(16) $(4n-5)(n^2-7n-2)$									
(17) $(3n-4)(4n^2+2n+3)$									
(18) $(n+8)(6n^2-n-4)$									
T	C	R	I	H	E	A	N	W	D
$6n^3+47n^2-12n-32$	$6n^3+44n^2-9n-32$	$4n^3-33n^2+27n+10$	$6n^3+10n^2+8n-4$	n^3+6n^2+9n-6	$12n^3-9n^2-2n-12$	$12n^3-10n^2+n-12$	n^3+7n^2+7n-6	$4n^3-30n^2+21n+10$	$12n^3+14n^2-4n+3$

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What Did the Girl Mushroom Say About the Boy Mushroom After Their First Date?

For each exercise below, multiply the polynomial by the monomial. Find your answer in the set of answers under the exercise and notice the letter next to it. Write this letter in the box that contains the number of that exercise.

(1) $5(2n^2+n)$

(2) $3n(8n^2-2n)$

(3) $n^2(4n-3)$

(4) $-2n(4+5n^3)$

(5) $-6n^2(4n^2-9)$

Answers:

(B) $-24n^4-54n$

(T) $24n^3-4n$

(R) $-24n^4+54n^2$

(U) $4n^3-3n^2$

(S) $10n^2+5n$

(L) $24n^3-6n^2$

(O) $-8n-6n^3$

(A) $-8n-10n^4$

(6) $4a(a^2-2a+3)$

(7) $-2a^2(9-a-4a^2)$

(8) $a^2b(a^2-b^2)$

(9) $-3ab^2(a^3b^2-2a^2b)$

(10) $2ab(a^2+4ab-3b^2)$

Answers:

(M) $4a^3-8a^2+10$

(H) $-18a^2+2a^3+8a^4$

(E) $2a^3b+8a^2b^2-6ab^3$

(I) $2a^3b+8ab^2-4ab$

(A) $a^4b-a^2b^3$

(G) $4a^3-8a^2+12a$

(W) $-18a^2+2a^3+6a^5$

(L) $-3a^4b^4+6a^3b^3$

(11) $x^2y(2x^2-4xy+y^2)$

(12) $-2xy^2(2x^4-5x^2y^2-3y^4)$

(13) $4x^3y(-x^2y+2xy-5xy^2)$

(14) $-x^2y^3(7xy^3-x^2y^2+3x^3y)$

(15) $3x^2y^2(2x^4y^2-3x^2y-1)$

Answers:

(N) $-4x^5y^2+10x^3y^4+6xy^6$

(S) $2x^4y-4x^2y^3+x^2y^4$

(E) $-4x^5y^2+8x^4y^2-20x^4y^3$

(U) $-4x^5y^2+10x^2y^4-20x^2y^3$

(Y) $2x^4y-4x^3y^2+x^2y^3$

(F) $6x^6y^4-9x^4y^3-3x^2y^2$

(T) $-7x^3y^6+x^5y^4-3x^3y^4$

(I) $-7x^3y^6+x^4y^5-3x^5y^4$

7	10	1	5	13	4	9	2	11	8	15	3	12	6	14

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Where Do Tadpoles in the Pawn Shop Come From?

Factor each polynomial below as the product of its greatest monomial factor and another polynomial. Find your answer and notice the letter next to it. Write this letter in each box that contains the number of that exercise.

(1) $3x^2+18x+9$

(2) $2x^2+10x+12$

(3) $7x^2+14x+35$

(4) $5x^2-20x+10$

(5) $6x^2+9x-21$

Answers:

(D) $3(2x^2+3x-7)$

(L) $3(2x^2+4x-5)$

(A) $3(x^2+6x+3)$

(P) $5(x^2-2x+5)$

(F) $5(x^2-4x+2)$

(O) $2(x^2+5x+6)$

(B) $7(x^2+x+6)$

(E) $7(x^2+2x+5)$

(6) n^3+n^2+n

(7) $n^4-n^3+n^2$

(8) $2n^3-n^2-5n$

(9) $3n^2+9n$

(10) $7n^2-28n$

Answers:

(S) $n(2n^2-2n-6)$

(O) $n^2(n^2-n+1)$

(I) $7n(n+5)$

(F) $3n(n+3)$

(E) $n^2(n^2-2n+3)$

(A) $n(n^2+n+1)$

(M) $n(2n^2-n-5)$

(R) $7n(n-4)$

(11) $4k^3-32k$

(12) $6k^3+10k^2$

(13) $5k^3+15k^2+10k$

(14) $4k^3-20k^2+4$

(15) $4k^4+18k^3-6k^2$

Answers:

(P) $4k(k^3-5k^2+1)$

(R) $5k(k^2+3k+2)$

(S) $4(k^3-8k^2+2)$

(G) $4k(k^2-8)$

(L) $5k(k^2+4k+1)$

(W) $2k^2(2k^2+9k-3)$

(T) $2k^2(3k-9)$

(N) $2k^2(3k+5)$

4	10	2	8	1	9	13	7	11	14	6	15	12	3	5

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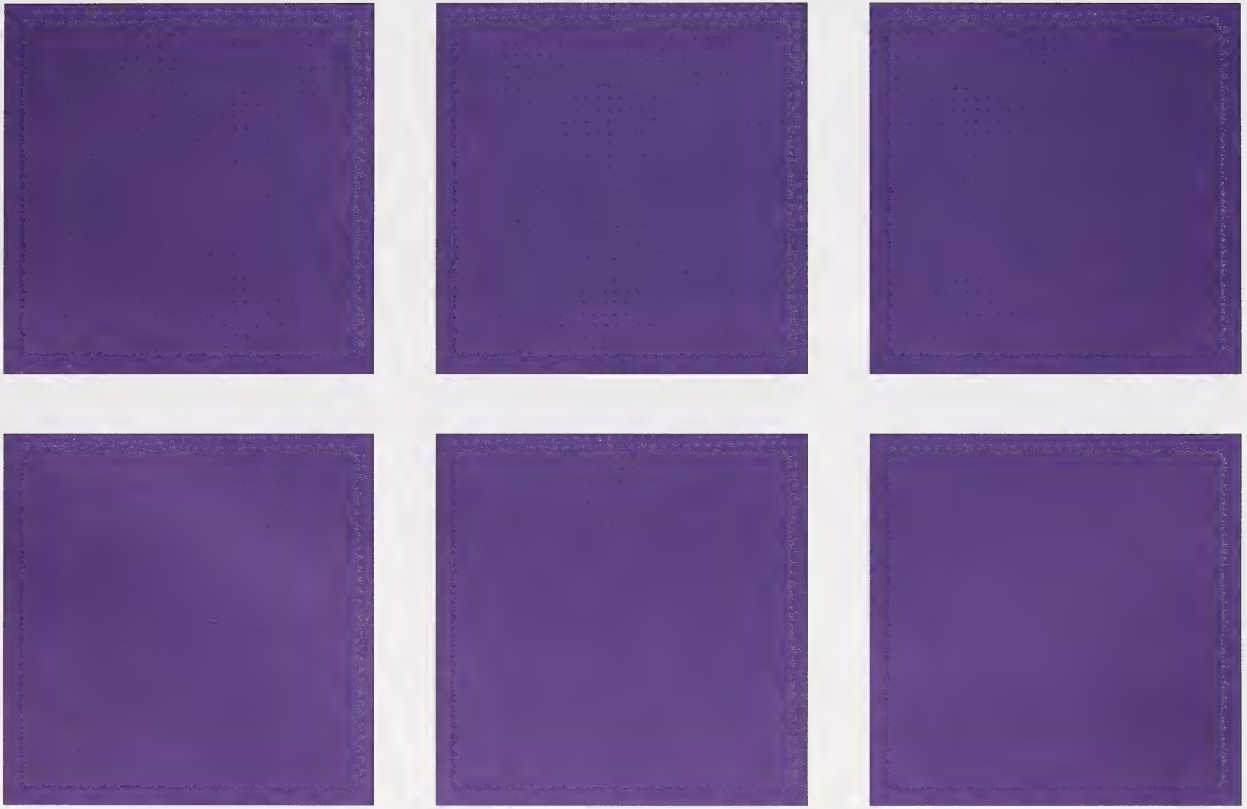
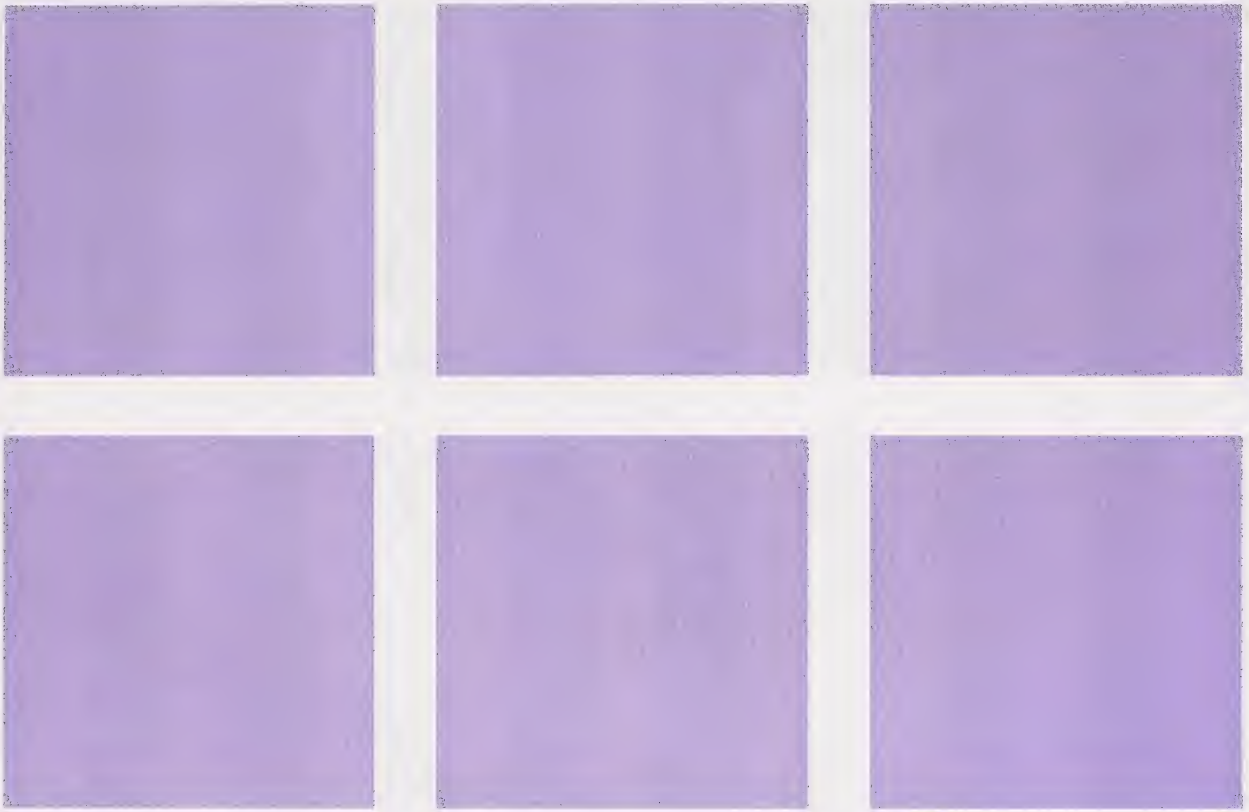
Cut-out Learning Aids

Algebra Tiles

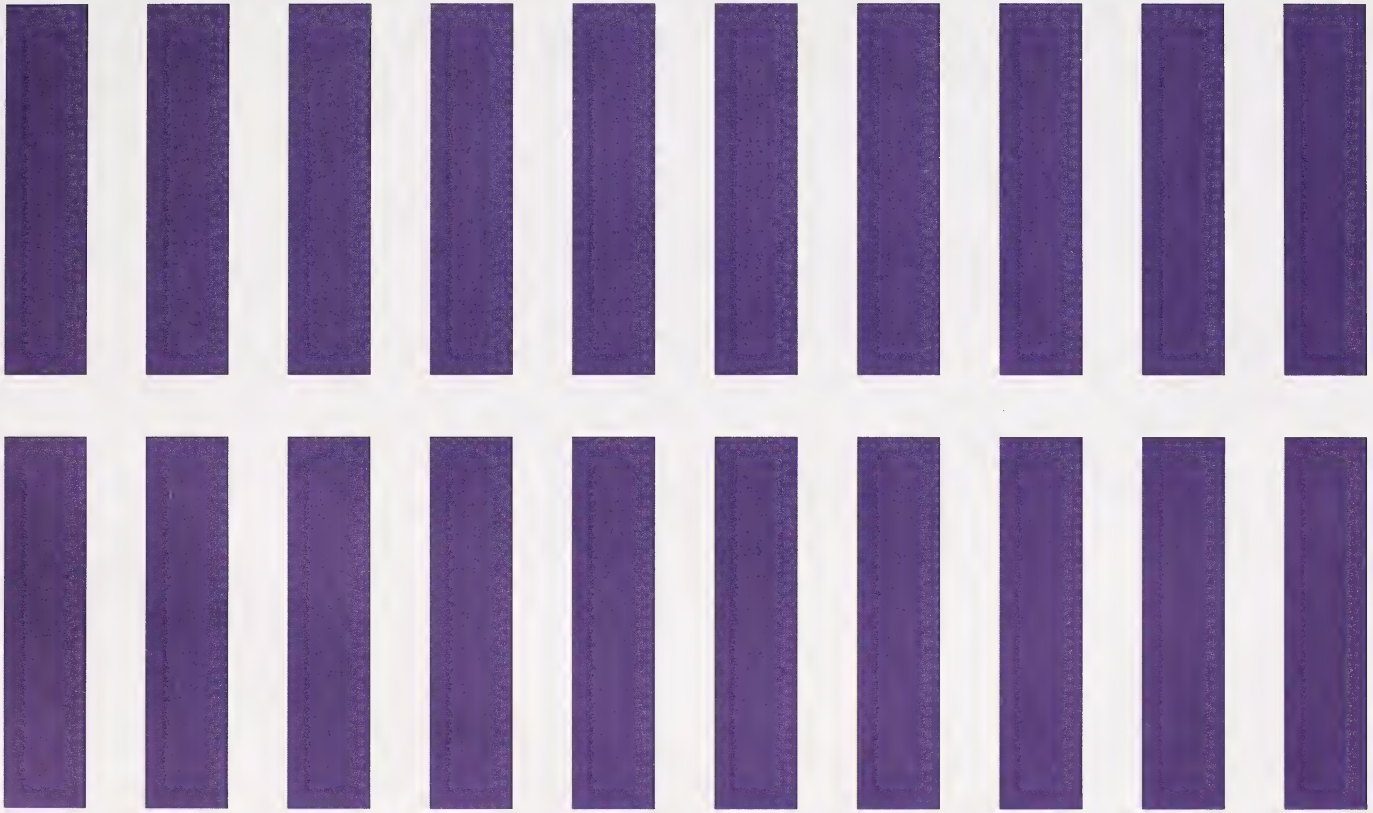
Each of these squares represents x^2.



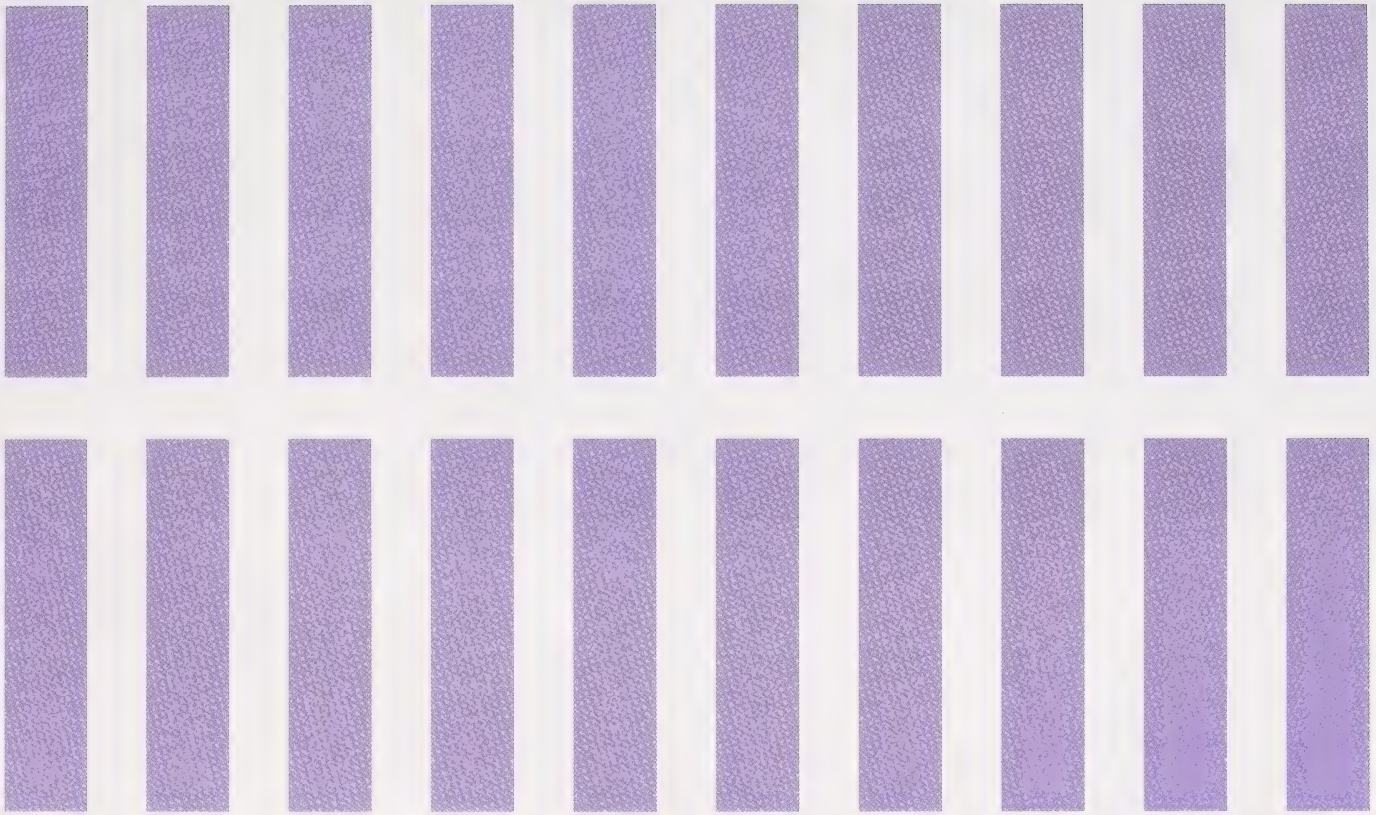
Each of these squares represents $-x^2$.



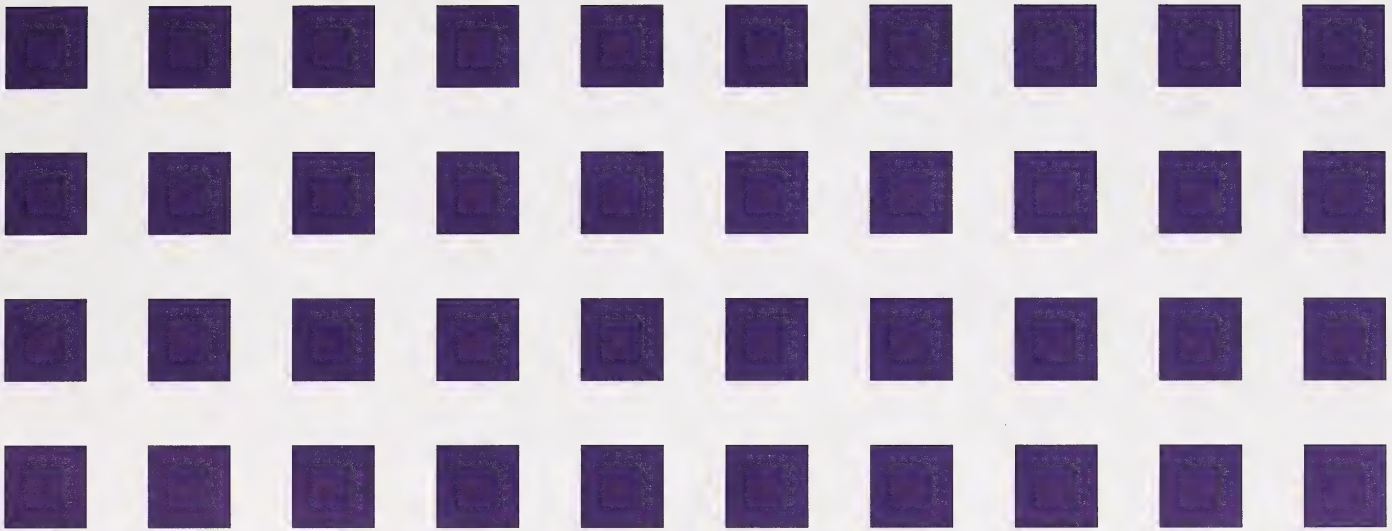
Each of these rectangles represents x.



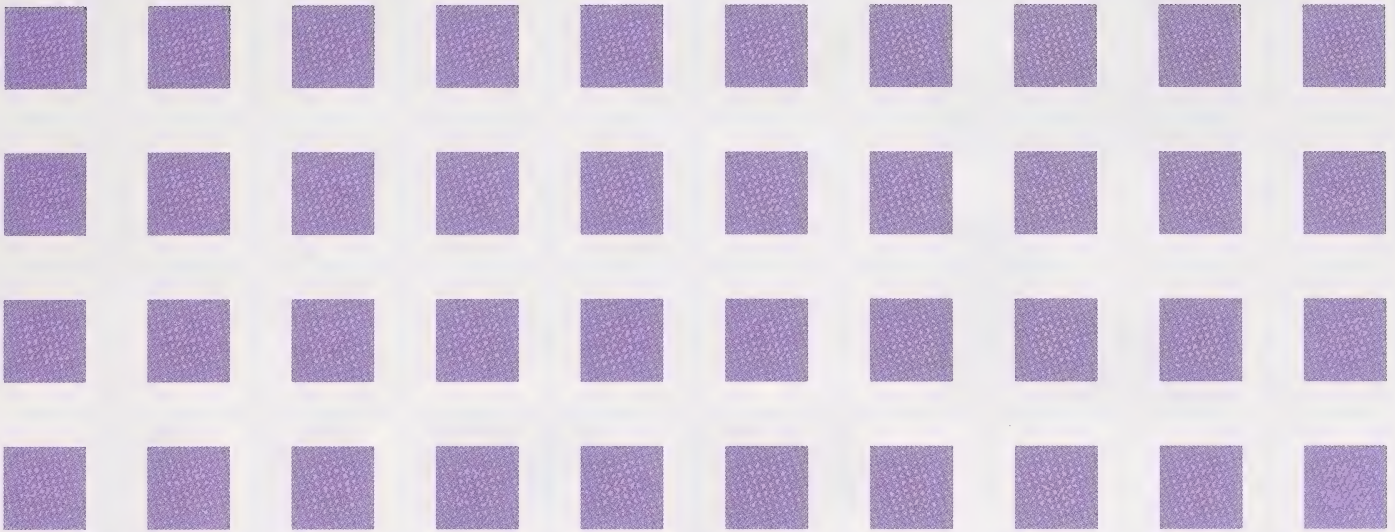
Each of these rectangles represents $-x$.



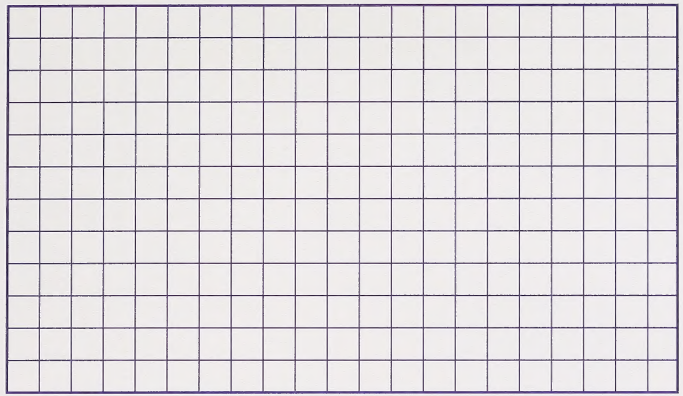
Each of these squares represents 1.

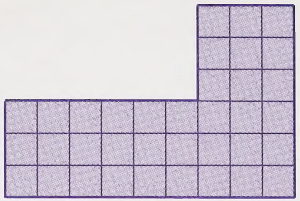


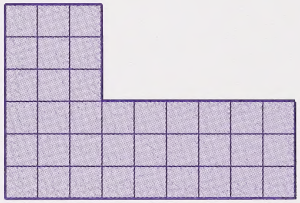
Each of these squares represents -1.

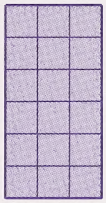


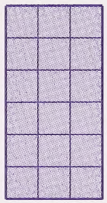
Grids and Pieces for Section 2: Activity 3

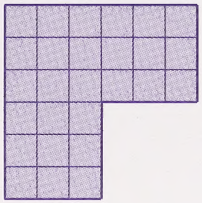


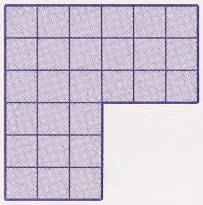


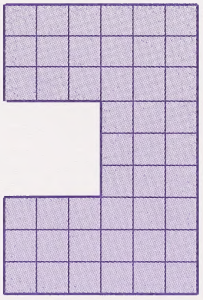


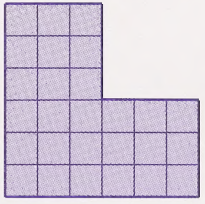












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